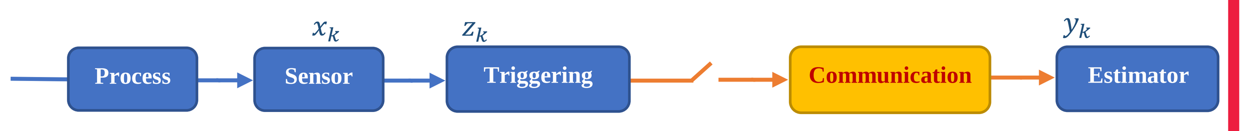
EVENT-BASED PARTICLE FILTERING WITH POINT AND SET-VALUED MEASUREMENTS

SOMAYEH DAVAR & ARASH MOHAMMADI
CONCORDIA INSTITUTE FOR INFORMATION SYSTEM ENGINEERING, MONTREAL, QC, CANADA

MOTIVATIONS/OBJECTIVES

Block diagram of the open-loop event-based estimation.



Motivation:

- The urgent need to develop, design, and implement intelligent transmission, scheduling, and estimation schemes to cope with recent/rapid growth of Cyber-Physical Systems (CPS) and Internet of Things (IoT) is the main motivation behind this research work.

Objective:

- Develop a mechanism to jointly incorporate point and set-valued measurements within the Particle Filtering framework.

Contributions:

- We propose an **Event-Based Particle Filtering(EBPF)**, for distributed state estimation problems where the remote sensor communicates its measurements to the fusion centre (FC) in an event-based fashion.

PROBLEM FORMULATION

We consider an estimation problem represented by the following linear state-space model

$$\begin{aligned} \boldsymbol{x}_k &= \boldsymbol{F}_k \boldsymbol{x}_{k-1} + \boldsymbol{w}_k \\ \boldsymbol{z}_k &= \boldsymbol{H}_k \boldsymbol{x}_k + \boldsymbol{v}_k, \end{aligned}$$

- The non-linear estimator, resided at the FC, jointly incorporates point and set-valued measurements to estimate the non-Gaussian posterior distribution.
- Due to joint incorporation of set and point valued measurements, the posterior distribution becomes **non-Gaussian**, therefore, the conventional Kalman Filter is not applicable.
- In an EB communication/fusion framework, after making each measurement the sensor decides on **keeping or sending its observation** to the remote estimator.
- The local decisions are governed by a binary triggering criteria denoted by γ_k which is defined as follows

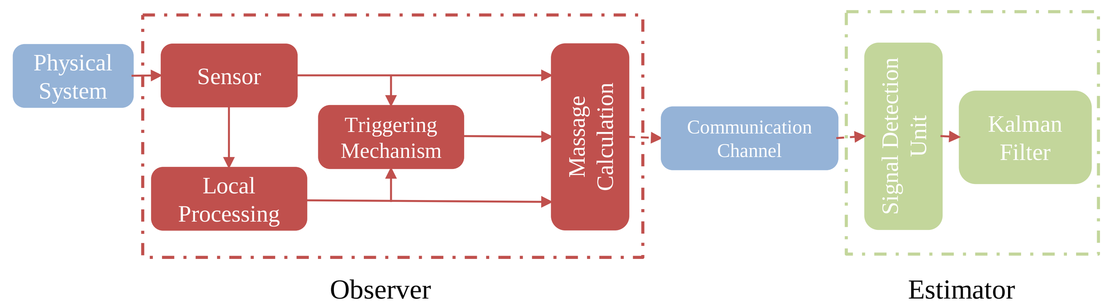
$$\begin{cases} \gamma_k = 1: & \text{Event occurs, communication is triggered.} \\ \gamma_k = 0: & \text{Idle case, no communication.} \end{cases}$$

- The EBPF implements the filtering recursions by propagating N_s particles $\mathbb{X}_k^i$ and associated weights as follows

$$\begin{aligned} \mathbb{X}_k^i &\sim q(\mathbb{X}_k^i | \mathbb{X}_{k-1}^i, \boldsymbol{Y}_{k-1}) \\ W_k^i &\propto W_{k-1}^i \frac{P(\boldsymbol{y}_k | \mathbb{X}_k^i) P(\mathbb{X}_k^i | \mathbb{X}_{k-1}^i)}{q(\mathbb{X}_k^i | \mathbb{X}_{k-1}^i, \boldsymbol{Y}_k)}. \end{aligned}$$

The 25th European Signal Processing Conference (EUSIPCO), Kos island, Greek, Aug 2017.

EVENT-BASED STATE ESTIMATION



The sensor computes the distance between its current measurement and the previously transmitted measurement based on the following event-triggering schedule

$$\gamma_k = \begin{cases} 1, & \text{if } |\boldsymbol{z}_k - \boldsymbol{z}_{\tau_k}| \geq \Delta \\ 0, & \text{otherwise}, \end{cases},$$

where τ_k denotes the time of last communication from the sensor to the FC, and Δ denotes the triggering threshold. Based on the above triggering mechanism, we define the hybrid observation vector as $\boldsymbol{Y}_k = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_k\}$ where

$$\boldsymbol{y}_k = \begin{cases} \boldsymbol{z}_k & \text{if } \gamma = 1 \\ \{\boldsymbol{z}_k : \boldsymbol{z}_k \in (\boldsymbol{z}_{\tau_k} - \Delta, \boldsymbol{z}_{\tau_k} + \Delta)\} & \text{if } \gamma = 0 \end{cases}.$$

(i) Update based on Set-valued Measurements ($\gamma_k = 0$)

- In the absence of the sensor measurement and by incorporation of side information, the likelihood function can be specified as follows

$$P(\boldsymbol{y}_k | \boldsymbol{x}_k, \gamma_k = 0) = P(\boldsymbol{z}_{\tau_k} - \Delta \leq \boldsymbol{z}_k \leq \boldsymbol{z}_{\tau_k} + \Delta),$$

(ii) Update based on Point Measurements ($\gamma_k = 1$):

- In this case, the estimator receives the sensor measurement $\boldsymbol{z}_k$, therefore, the hybrid likelihood function reduces to the sensor likelihood function given by

$$P(\boldsymbol{y}_k | \boldsymbol{x}_k, \gamma_k = 1) = P(\boldsymbol{z}_k | \boldsymbol{x}_k) = \Phi\left(\frac{\boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{x}_k}{\sqrt{R_k}}\right).$$

THE EBPF ALGORITHM

Input: $\{\mathbb{X}_{k-1}^{(i)}, W_{k-1}^{(i)}\}_{i=1}^{N_s}$, γ_k, and $\boldsymbol{y}_k$.

Output: $\{\mathbb{X}_k^{(i)}, W_k^{(i)}\}_{i=1}^{N_s}$, $\hat{\boldsymbol{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$.

At iteration k, EBPF updates its particle set as follows:

S1. *Predictive Particle Generation*: Sample *predicted particle* from the proposal distribution i.e., $\mathbb{X}_k^{(i)} \sim P(\boldsymbol{x}_k | \boldsymbol{x}_{k-1})$.

S2. *Hybrid Likelihood Computation*:

- **If** $\gamma_k = 0$: Compute $P(\boldsymbol{y}_k | \mathbb{X}_k^{(i)})$ as follows

$$\begin{aligned} &P(\boldsymbol{y}_k | \boldsymbol{x}_k, \gamma_k = 0) \\ &= \frac{1}{\sqrt{2\pi R_k}} \int_{\boldsymbol{z}_\tau - \Delta - \boldsymbol{H}_k \boldsymbol{x}_k}^{\boldsymbol{z}_\tau + \Delta - \boldsymbol{H}_k \boldsymbol{x}_k} \exp\left\{\frac{-t}{2R_k}\right\} dt \\ &= \Phi\left(\frac{\boldsymbol{z}_\tau + \Delta - \boldsymbol{H}_k \boldsymbol{x}_k}{\sqrt{R_k}}\right) - \Phi\left(\frac{\boldsymbol{z}_\tau - \Delta - \boldsymbol{H}_k \boldsymbol{x}_k}{\sqrt{R_k}}\right), \end{aligned}$$

- **If** $\gamma_k = 1$: Compute $P(\boldsymbol{y}_k | \mathbb{X}_k^{(i)})$ as follows

$$P(\boldsymbol{y}_k | \boldsymbol{x}_k, \gamma_k = 1) = P(\boldsymbol{z}_k | \boldsymbol{x}_k) = \Phi\left(\frac{\boldsymbol{z}_k - \boldsymbol{H}_k \boldsymbol{x}_k}{\sqrt{R_k}}\right).$$

MONTE-CARLO SIMULATIONS

Experimental Setup:

- A tracking problem is considered where Target's dynamic is given by

$$\boldsymbol{x}_k = \begin{bmatrix} 0.8 & 1 \\ 0 & 0.95 \end{bmatrix} \boldsymbol{x}_{k-1} + \boldsymbol{w}_k,$$

- The sensor periodically measures the position and speed of the target based on the following observation model

$$\boldsymbol{z}_k = \begin{bmatrix} 0.4 & 0.3 \end{bmatrix} \boldsymbol{x}_k + v_k.$$

- Four estimators are implemented and compared for accuracy:

(i) The full-rate estimation based on KF;
(ii) The full-rate estimation based on particle filter;
(iii) Open-loop and event-based KF with SOD triggering;
(iv) The proposed EBPF;

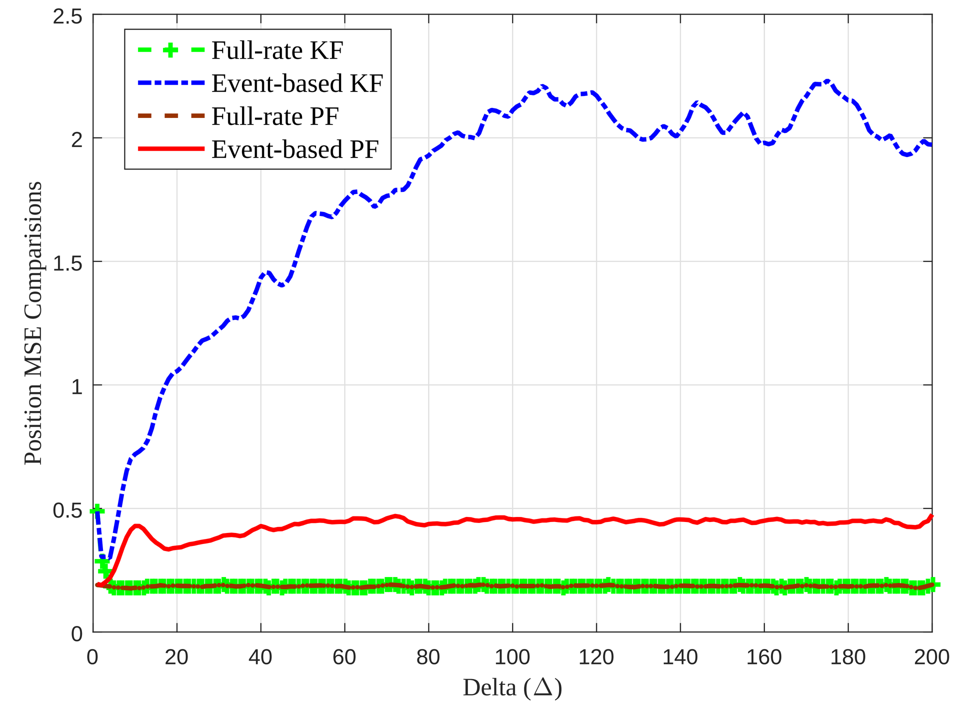


Figure 1: The Position MSE comparison when $\Delta = 1.2$.

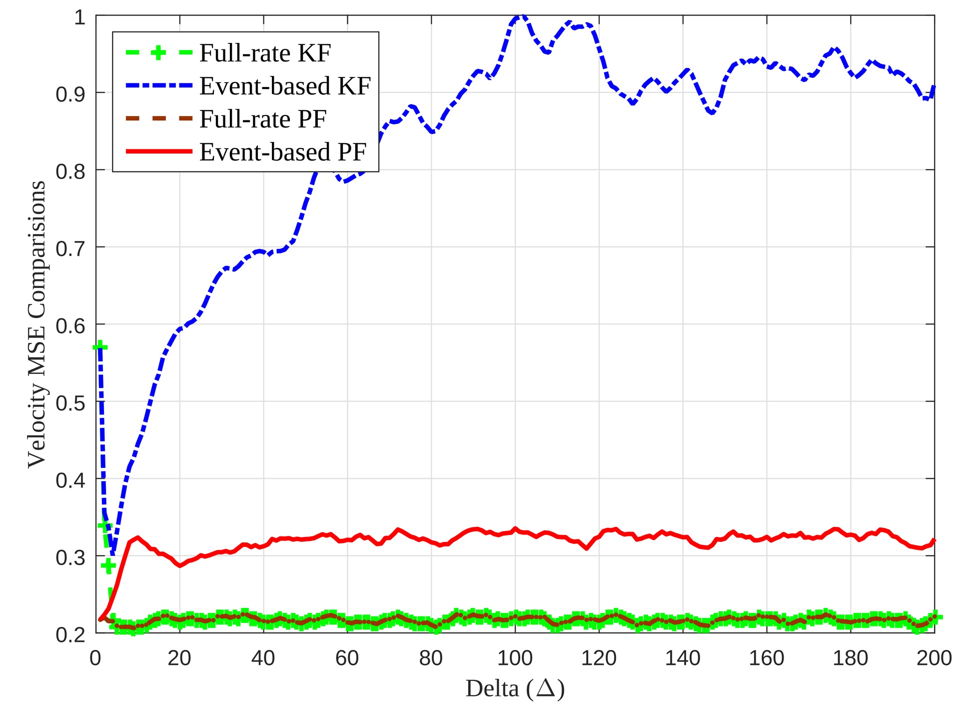


Figure 2: The Velocity MSE comparison when $\Delta = 1.2$.

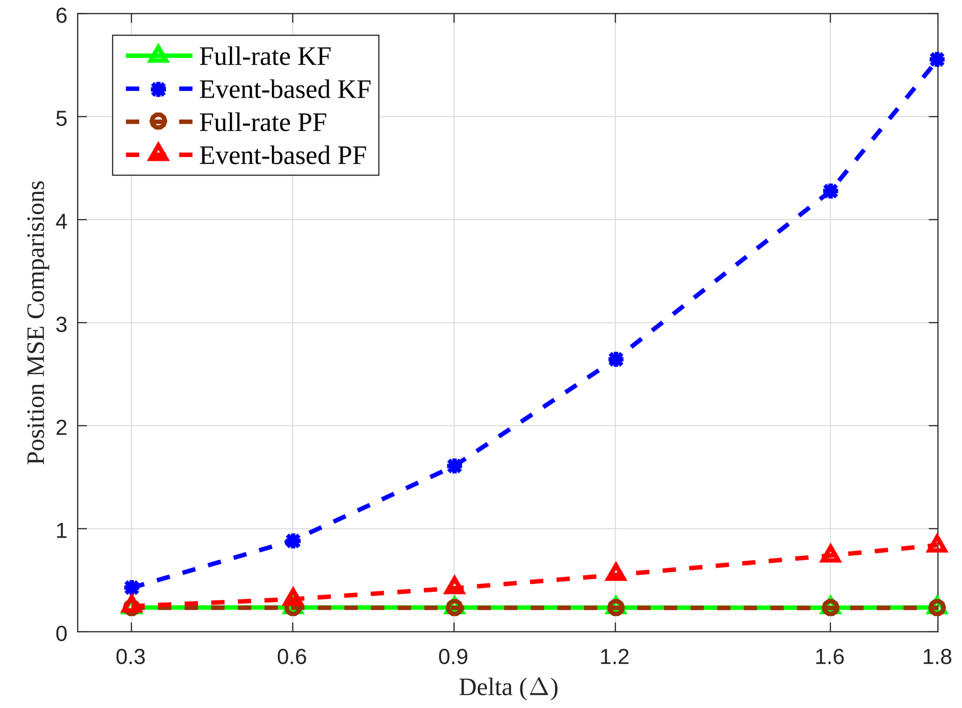


Figure 3: Position MSE over different values of Δ.

- The proposed EBPF algorithm provides acceptable results in very low communication rates (high values of Δ) and closely follows its full-rate counterparts in high communication rates. Besides, when the communication rate increases (i.e., small values for Δ), the proposed event-based methodology approaches the full-rate estimator.

CONCLUSION

Summary:

- We proposed an ***Event-based Particle Filter (EBPF) Framework***, for distributed state estimation in systems with communication/power constraints at the sensor side.
- Our main contribution is proposing the EBPF which jointly incorporates ***Point and Set-Valued Measurements*** within the particle filter framework.
- In presence of an observation ***(Point-Valued Measurement)***, the likelihood function can exactly be evaluated for each particle.
- In the absence of an observation, ***Set-Valued Measurement***, the likelihood becomes the probability that the observation belongs to the triggering set.

REFERENCES

[1] A. Mohammadi and K. N. Plataniotis, "Event-Based Estimation With Information-Based Triggering and Adaptive Update," *IEEE Trans. Signal Process.*, vol. 65, no. 18, pp. 4924-4939, Sept.15, 15 2017.

[2] A. Mohammadi, A. Asif, Distributed Consensus + Innovation Particle Filtering for Bearing/Range Tracking With Communication Constraints," *IEEE Trans. Signal Process.*, vol. 63, no. 3, pp. 620-635, 2015.