Distributed Inference and Management of Future Cyber-Physical Networks

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SMART GRID: Advanced infrastructure and information technologies (**Cyber**) to enhance the electrical power network (**Physical**)

controllable

resilient

efficient

participation

sustainable

self-restoring

green

situational awareness

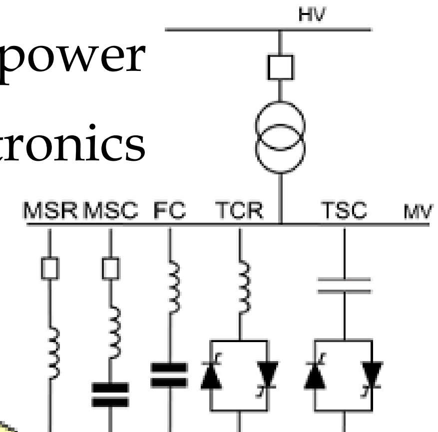
Dept. of Energy, "The smart grid: an introduction"

Enabling technology advances

renewables

power

electronics



distributed generation

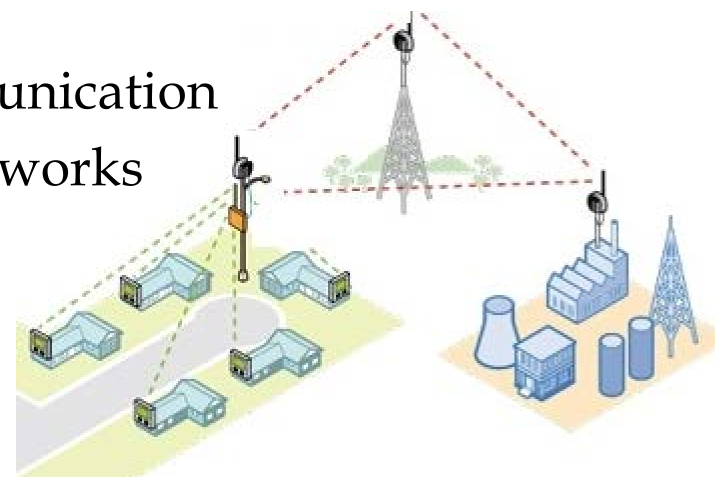
micro-grids

Optimization, learning, and signal processing toolbox

electric vehicles

sensing/metering

communication networks



demand response

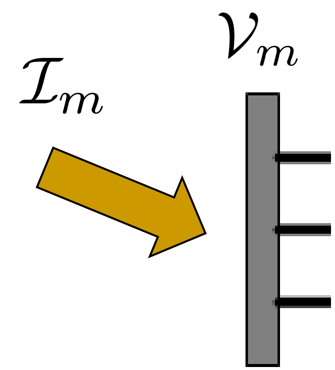
Outline

- Distributed and robust
  power system state estimation (PSSE)
- Distributed optimal power flow (OPF)
- Distributed demand response (DR)
- Distributed electric vehicle (EV) charging

Complex power

- Power injection to bus *m*

$$\mathcal{S}_m = \mathcal{V}_m \mathcal{I}_m^* = P_m + jQ_m$$



- (Re) active power generated or consumed at a bus

- Power flow over line (*m*, *n*) $\quad \mathcal{S}_{mn} = \mathcal{V}_m \mathcal{I}_{mn}^* = P_{mn} + jQ_{mn}$

- Multivariate nodal power model (*quadratic* in **v**)

concatenating $\{\mathcal{V}_m\}$ — bus admittance matrix

$$\mathbf{s} = \text{diag}(\mathbf{v})\mathbf{i}^* = \text{diag}(\mathbf{v})\mathbf{Y}^*\mathbf{v}^*$$

concatenating $\{\mathcal{S}_m\}$ — concatenating $\{\mathcal{I}_m\}$

Power system state estimation

Motivation for PSSE

Goal: Given meter readings and grid parameters, find state vector $\mathbf{v}$

- Quantities of interest expressible as functions of bus voltages in $\mathbf{v}$
- PSSE is of paramount importance for
  - Situational awareness
  - Reliability analysis and planning
  - Load forecasting
  - Economic operations and billing
- Can be formulated as an estimation problem [Schweppe et al'70]

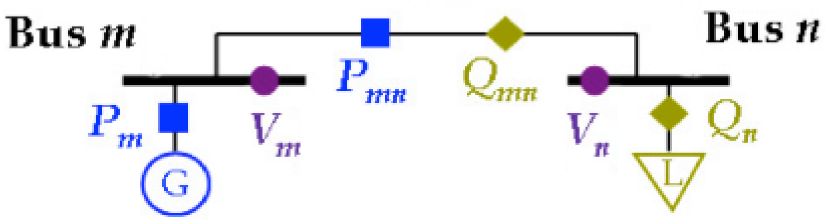
F. C. Schweppe, J. Wildes, and D. Rom, "Power system state estimation: Parts I, II, and III," *IEEE Trans. Power App. Syst.*, Jan. 1970.

SCADA-based PSSE

- Supervisory control and data acquisition (SCADA) system
  - Terminals forward readings to control center (~4 secs)
  - Phases cannot be used due to timing mismatches
- Available measurements (M)

  $\{V_m, P_m, Q_m, P_{mn}, Q_{mn}, I_{mn}\}$

  $\mathbf{z} = \mathbf{h}(\mathbf{v}) + \boldsymbol{\epsilon}$



- Nonlinear (weighted) least-squares

$$\hat{\mathbf{v}} := \arg\min_{\mathbf{v}} \|\mathbf{z} - \mathbf{h}(\mathbf{v})\|^2$$

- Constraints
  - Zero-injection buses $P_m = Q_m = 0$
  - Feasible ranges $V_m^{\min} \leq V_m \leq V_m^{\max}$

Popular solvers

$$\hat{\mathbf{v}} := \arg\min_{\mathbf{v}} \|\mathbf{z} - \mathbf{h}(\mathbf{v})\|^2$$

(M1) Gauss-Newton iterations

- Approximate $\mathbf{h}(\mathbf{v}) \simeq \mathbf{h}(\mathbf{v}_k) + \mathbf{G}_k^T(\mathbf{v} - \mathbf{v}_k),\ \mathbf{G}_k\ :\ \text{Jacobian at } \mathbf{v}_k$
- Linear LS in closed form $\mathbf{v}_{k+1} = \mathbf{v}_k + (\mathbf{G}_k\mathbf{G}_k^T)^{-1}\mathbf{G}_k(\mathbf{z} - \mathbf{h}(\mathbf{v}_k))$

- Cholesky factorization based remedies for numerical stability
- Sensitive to initialization; No convergence guarantee

(M2) Fast decoupled solver

- Active powers depend only on $\{\theta_m\}$; reactive only on $\{V_m\}$
- Approximate $(\mathbf{G}_k\mathbf{G}_k^T)^{-1}$ at *flat voltage profile* $\mathbf{v} = \mathbf{1} + j\mathbf{0}$

A. Monticelli and A. Garcia, "Fast decoupled state estimators," *IEEE Trans. Power Syst.*, May 1990.

Semidefinite relaxation

- Rectangular coordinates: measurements are ***quadratic*** in $\mathbf{v}$

$$P_m + jQ_m = \mathcal{V}_m \mathcal{I}_m^* = \mathbf{e}_m^T \mathbf{v} (\mathbf{Y}\mathbf{v})^H \mathbf{e}_m = \mathrm{Tr}(\underbrace{\mathbf{Y}^H \mathbf{e}_m \mathbf{e}_m^T}_{\mathbf{H}_m} \mathbf{v}\mathbf{v}^H)$$

- Yet ***linear*** in $\mathbf{V} = \mathbf{v}\mathbf{v}^H$

$$\min_{\mathbf{v}} \sum_{m=1}^{M} (z_m - h_m(\mathbf{v}))^2 \quad \Rightarrow \quad \min_{\mathbf{V}} \sum_{m=1}^{M} (z_m - \mathrm{Tr}(\mathbf{H}_m \mathbf{V}))^2$$

$$\text{s.to} \quad \mathbf{V} \succeq \mathbf{0} \text{ and } \cancel{\mathrm{rank}(\mathbf{V}) = 1}$$

- SDR popular in SP and communications [Goemans et al'95]
- SDR for SE [Zhu-GG'11], SDR for OPF [Bai etal'08], [Lavaei-Low'11]
  - Generalizations include PMU data, and robust SDR-based PSSE
  - (Near-)optimal regardless of initialization; polynomial complexity $O(N^{4.5} \log(1/\epsilon))$

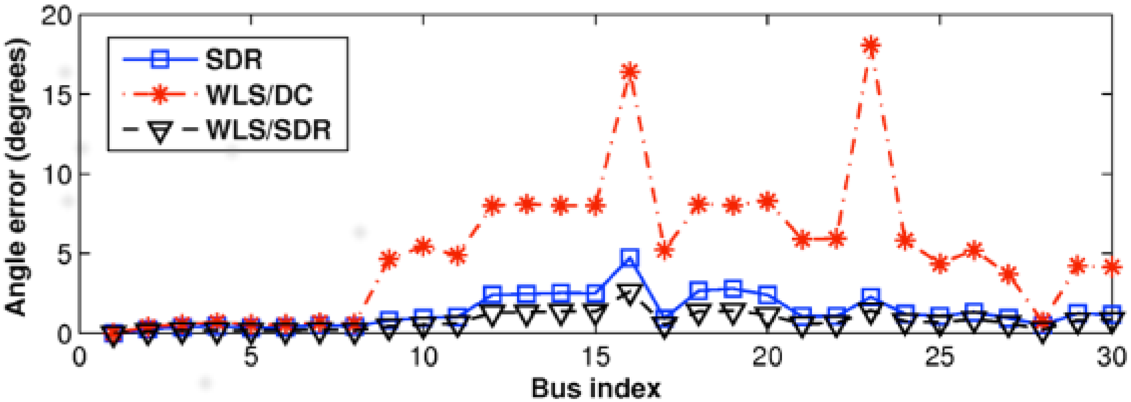
H. Zhu and G. B. Giannakis, "Estimating the state of AC power systems using semidefinite programming," in *Proc. of North American Power Symposium*, Aug. 2011.

Numerical tests

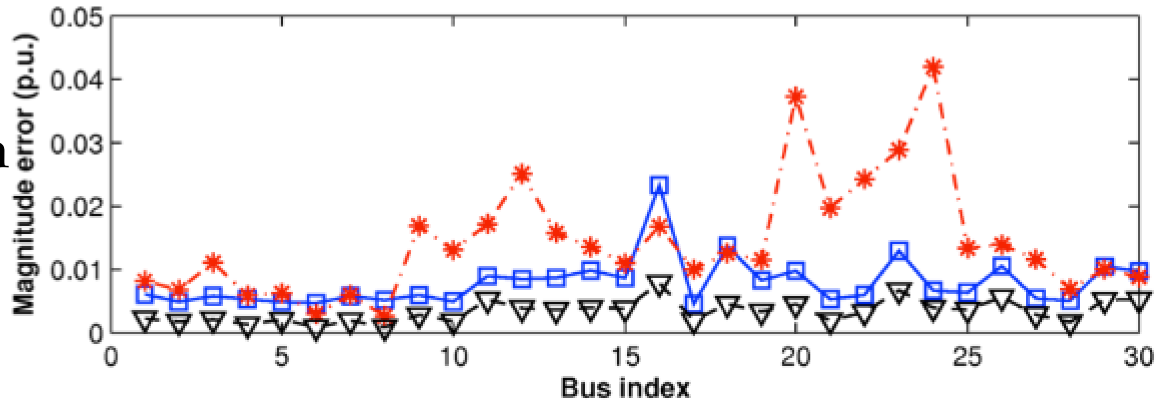
- IEEE 30, 57, and 118-bus benchmarks
- $V_m \sim \mathcal{N}(1, 0.01),\ \theta_m \sim \mathcal{U}[-\theta, \theta]$

Average running time in secs.

# of buses	WLS	SDR
30	0.216	1.62
57	0.558	4.32
118	2.87	21.6

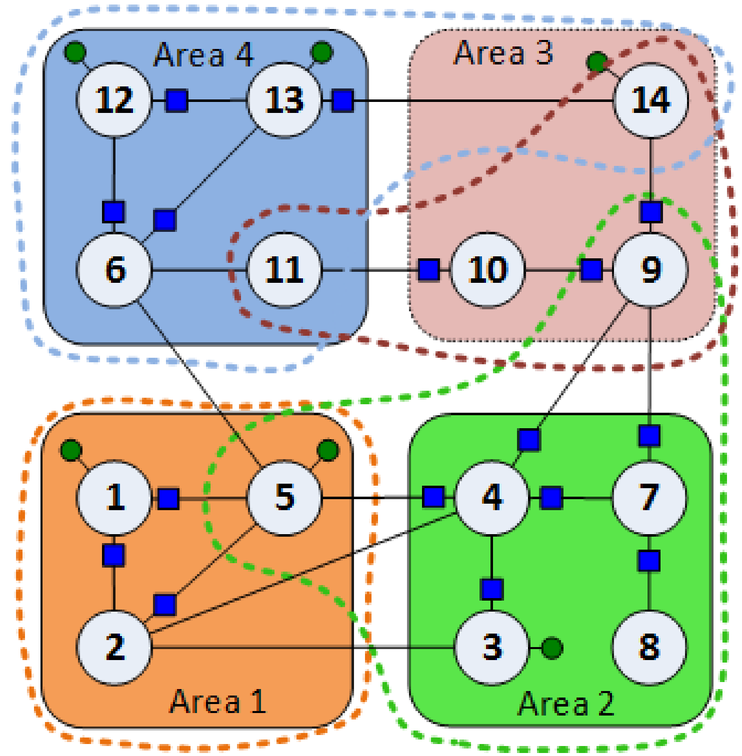


- Closer to global optimum at higher complexity



Decentralized PSSE - motivation

- Scalable with control area size, and privacy preserving
- Area 2 buses (states): *{3,4,7,8}*
- Area 2 collects flow measurements *{(4,5), (4,9), (7,9)}…*

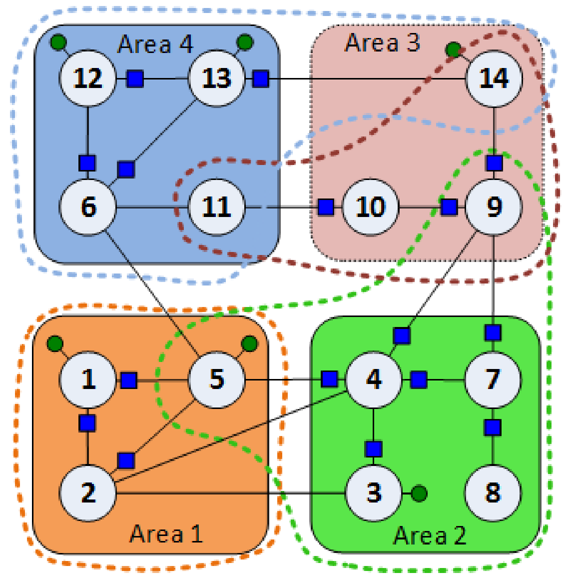


- *Option 1:* Ignore tie-line meters
  - statistically suboptimal
  - observability at risk (bus 11)
  - tie-line mismatches (trading)
- *Option 2:* Augment $\mathbf{v}_2$ to *{3,4,7,8,5,9}*
  - consent with neighbors on shared states

G. Korres, "Distributed multi-area state estimation," *IEEE Trans. Power Syst.*, vol. 26, no. 1, pp. 73-84, Feb. 2011.

Cost decomposition

- Include tie-line buses to split local LS cost per $\mathcal{N}_{(k)}$



$$f_k(\mathbf{V}_{(k)}) := \sum_{\ell=1}^{M_k} \left[z_k^\ell - \mathrm{Tr}(\mathbf{H}_{(k)}^\ell \mathbf{V}_{(k)}) \right]^2$$

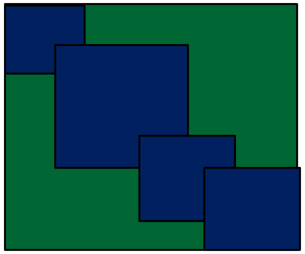
$\mathcal{N}_{(2)} := \mathcal{N}_2 \cup \{5, 9\}$

$$\hat{\mathbf{V}} := \arg\min_{\mathbf{V}} \sum_{k=1}^{K} \sum_{\ell=1}^{M_k} \left[z_k^\ell - \mathrm{Tr}(\mathbf{H}_k^\ell \mathbf{V}) \right]^2 \quad \text{s.to} \quad \mathbf{V} \succeq \mathbf{0}$$

$\rightarrow$

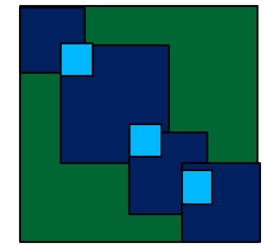
$$\hat{\mathbf{V}} := \arg\min_{\mathbf{V}} \sum_{k=1}^{K} f_k(\mathbf{V}_{(k)}) \quad \text{s.to} \quad \mathbf{V} \succeq \mathbf{0}$$

Challenge: as $\{\mathcal{N}_{(k)}\}$ overlap partially, PSD constraint couples $\{\mathbf{V}_{(k)}\}$



Blessing: overlap $\rightarrow$ global; no overlap: $\mathbf{V} \succeq \mathbf{0} \Leftrightarrow \mathbf{V}_{(k)} \succeq \mathbf{0}, \quad \forall k$

Distributed SDP for PSSE



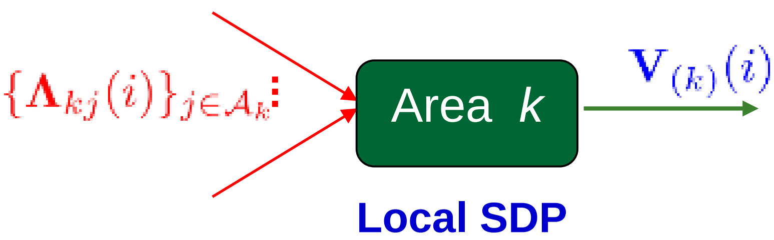
- If ***graph*** with *areas-as-nodes* and *overlaps-as-edges* is a ***tree***, then

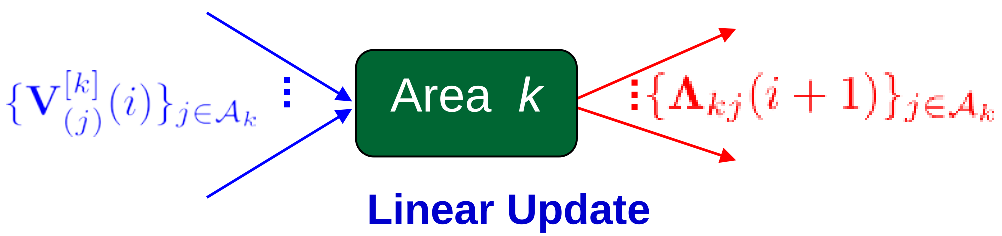
$$\hat{\mathbf{V}} := \arg\min_{\mathbf{V}} \sum_{k=1}^{K} f_k(\mathbf{V}_{(k)}) \quad \text{s.to} \quad \mathbf{V} \succeq \mathbf{0}$$

(C-SDP)

$$\{\hat{\mathbf{V}}_{(k)}\} := \arg\min_{\{\mathbf{V}_{(k)}\}} \sum_{k=1}^{K} f_k(\mathbf{V}_{(k)}) \quad \text{s.to} \quad \mathbf{V}_{(k)} \succeq \mathbf{0}, \quad \mathbf{V}_{(k)}^{[j]} = \mathbf{V}_{(j)}^{[k]}$$

- ADMM [Glowinski-Marrocco'75]; for D-Estimation [Schizas-Giannakis'07]
  - Iterates between local variables and multipliers per equality constraint

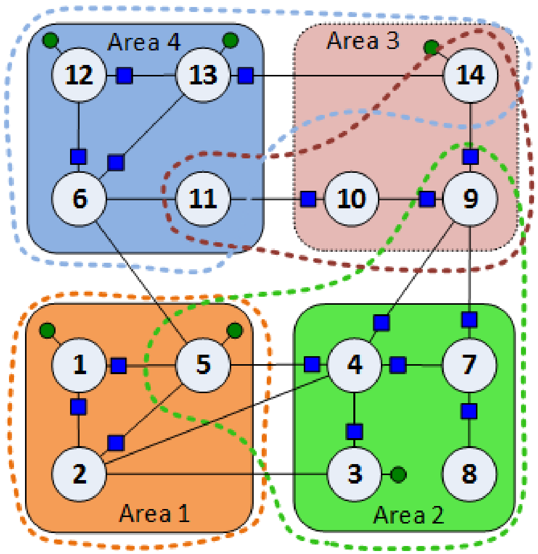


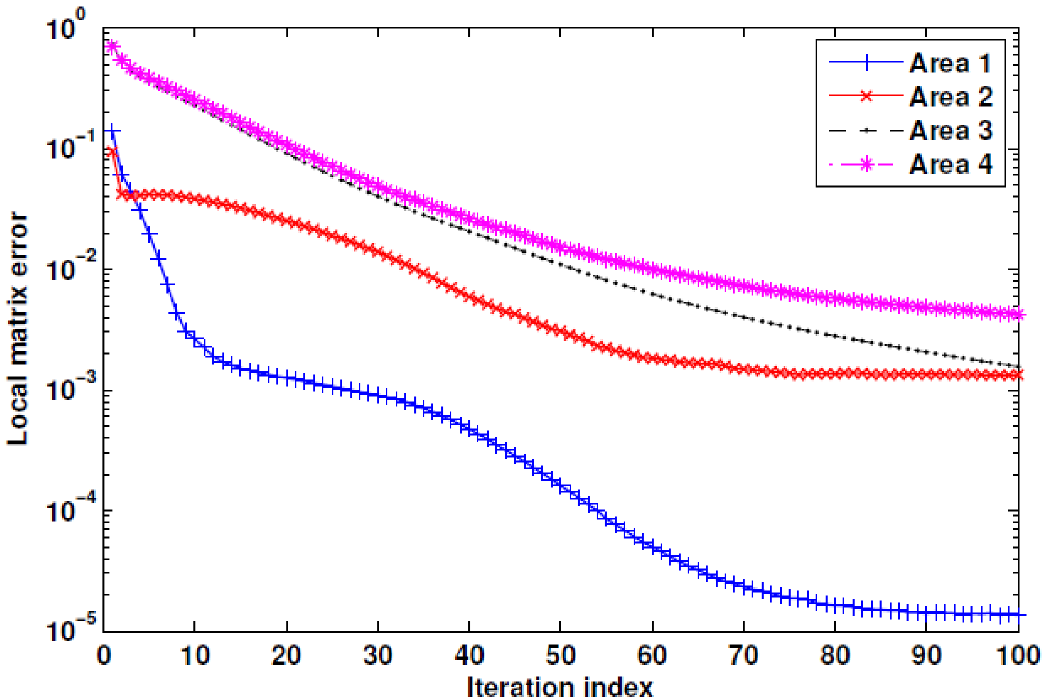


- Converges $\mathbf{V}_{(k)}(i) \rightarrow \hat{\mathbf{V}}_{(k)}$ even for noisy-async. links [Schizas-GG'08], [Zhu-GG'09]

ADMM convergence in action

- IEEE 14-bus grid with 4 areas; 5 meters on tie-lines



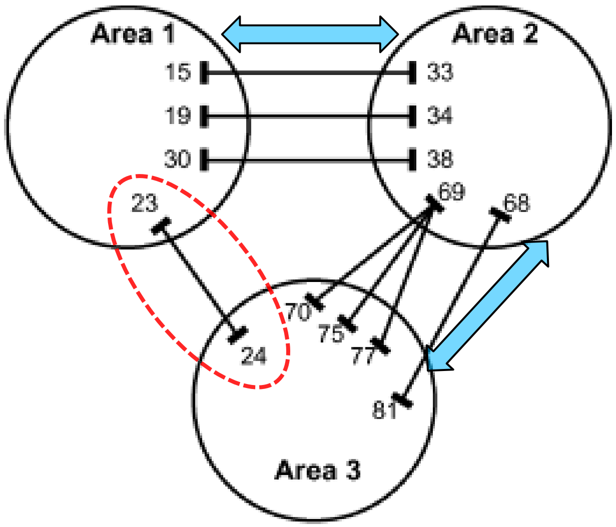


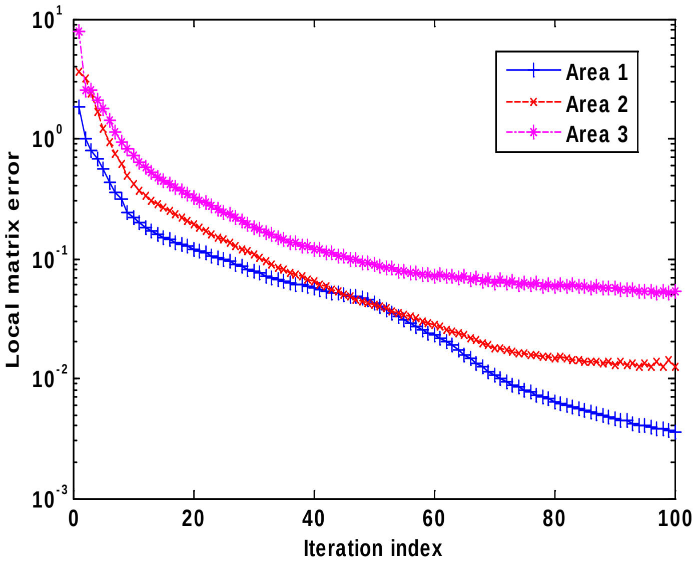
- Errors $\|\mathbf{V}_{(k)}(i) - \hat{\mathbf{V}}_{(k)}\|_F$ vanish asymptotically

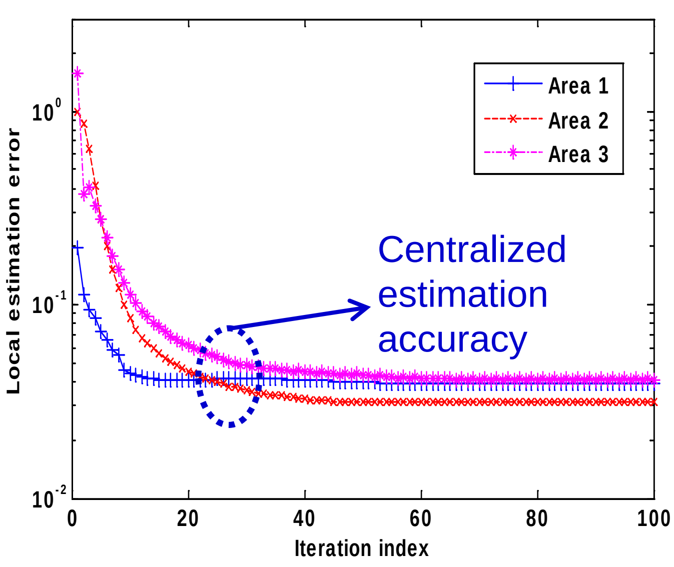
118-bus test case

- Triangular configuration [Min-Abur'06]
- Power flow meters on all tie lines except for (23, 24)

  ➡ graph of areas is a tree







- Local norms

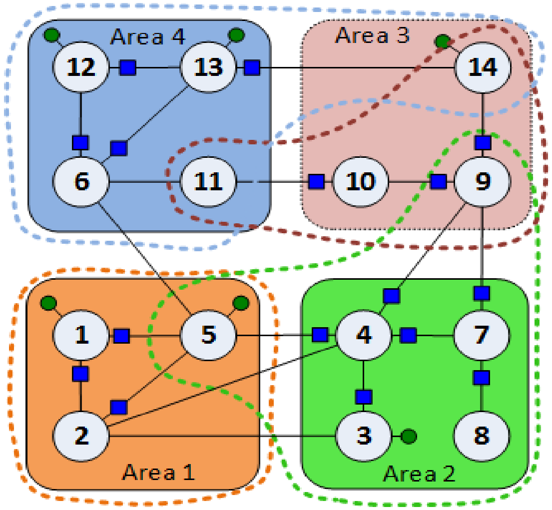
$$\|\mathbf{v}_{(k)}(i) - \mathbf{v}_{(k)}\|_2$$

converge in only 20 iterations!

H. Zhu and G. B. Giannakis, "Power system nonlinear state estimation using distributed semidefinite programming, *IEEE J. Sel. Topics Signal Process.*, pp. 1039-1050, Dec. 2014.

Decentralized PSSE for linear models

- Local linear(ized) model $\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k$
- Regional PSSEs $$\min_{\mathbf{x}_k \in \mathcal{X}_k} \; f_k(\mathbf{x}_k)$$
- Coupled local problems $$\min_{\{\mathbf{x}_k\}} \; \sum_{k=1}^{K} f_k(\mathbf{x}_k) \quad \text{s.t.} \quad \mathbf{x}_k[l] = \mathbf{x}_l[k]$$



$\mathcal{S}_2 := \mathcal{N}_{(2)} \setminus \mathcal{N}_2$

S1. $$\mathbf{x}_k^{t+1} \Leftarrow \arg \min_{\mathbf{x}_k \in \mathcal{X}_k} f_k(\mathbf{x}_k) + \frac{c}{2} \sum_{i \in \mathcal{S}_k} \left(x_k(i) - p_k^t(i)\right)^2$$

S2. $$p_k^{t+1}(i) \Leftarrow p_k^t(i) + \left(x_l^{t+1}[i] - \frac{x_k^t(i) + x_l^t[i]}{2}\right)$$

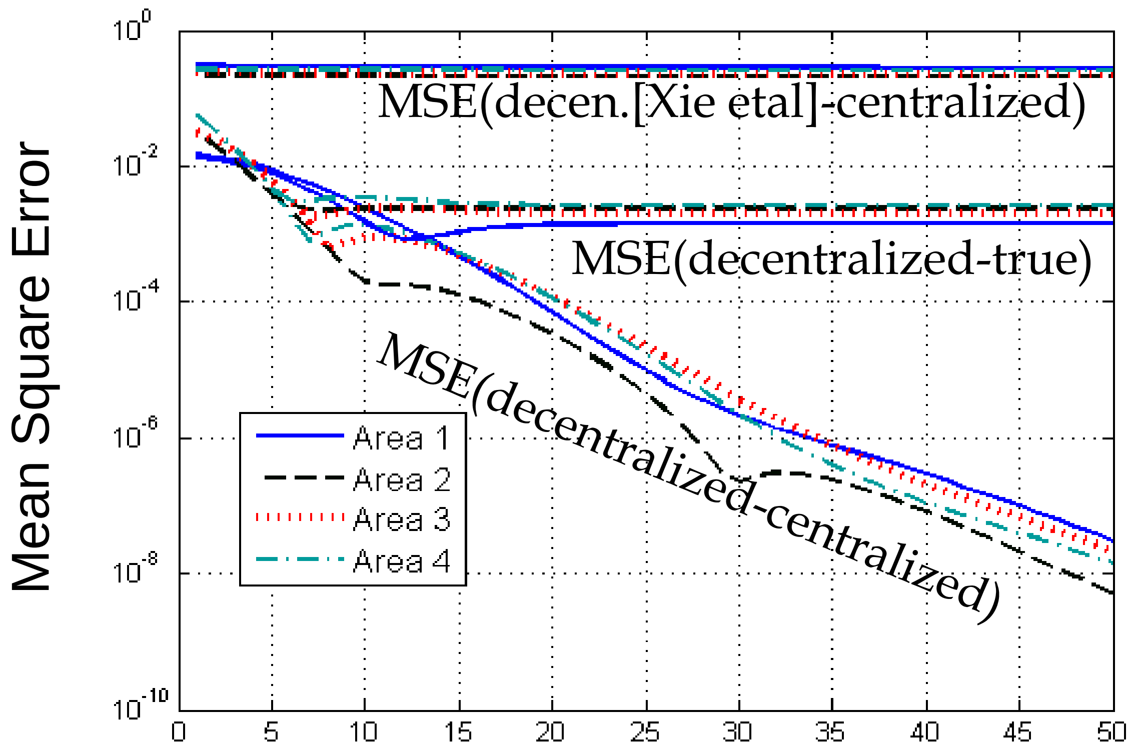
➢ ADMM solver: convergent with minimal exchanges and privacy-preserving

V. Kekatos and G. B. Giannakis, "Distributed robust power system state estimation," *IEEE Trans. Power Syst.*, vol. 28, pp. 1617-1626, May 2013.

Simulated test

S1. $\mathbf{x}_k^{t+1} \Leftarrow \left(\mathbf{H}_k^T\mathbf{H}_k + c\cdot\mathbf{D}_k\right)^{-1}\left(\mathbf{H}_k^T\mathbf{z}_k + c\cdot\mathbf{D}_k\mathbf{p}_k^t\right) \qquad [\mathbf{D}_k]_{ii} = |\mathcal{S}_k^i|$

S2. $p_k^{t+1}(i) \Leftarrow p_k^t(i) + \left(x_l^{t+1}[i] - \frac{x_k^t(i) + x_l^t[i]}{2}\right)$



L. Xie, C. Choi, and S. Kar, "Cooperative distributed state estimation: Local observability relaxed," *Proc. IEEE PES General Meeting*, Detroit, MI, July 2011.

Decentralized bad data cleansing

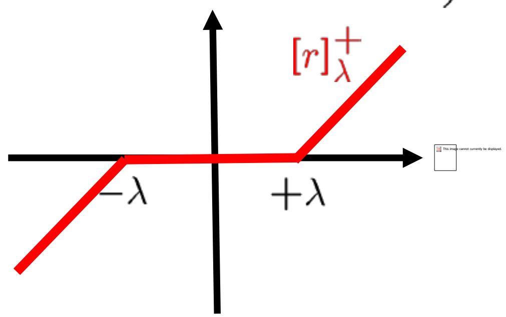
$$\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{n} + \mathbf{o}$$

- Reveal *single* and *block* outliers via

$$f(\mathbf{x}) := \min_{\mathbf{o}} \frac{1}{2}\|\mathbf{z} - \mathbf{H}\mathbf{x} - \mathbf{o}\|_2^2 + \lambda\|\mathbf{o}\|_1$$
$$= \sum_{m=1}^{M} h(z_m - \mathbf{h}_m^T\mathbf{x})$$

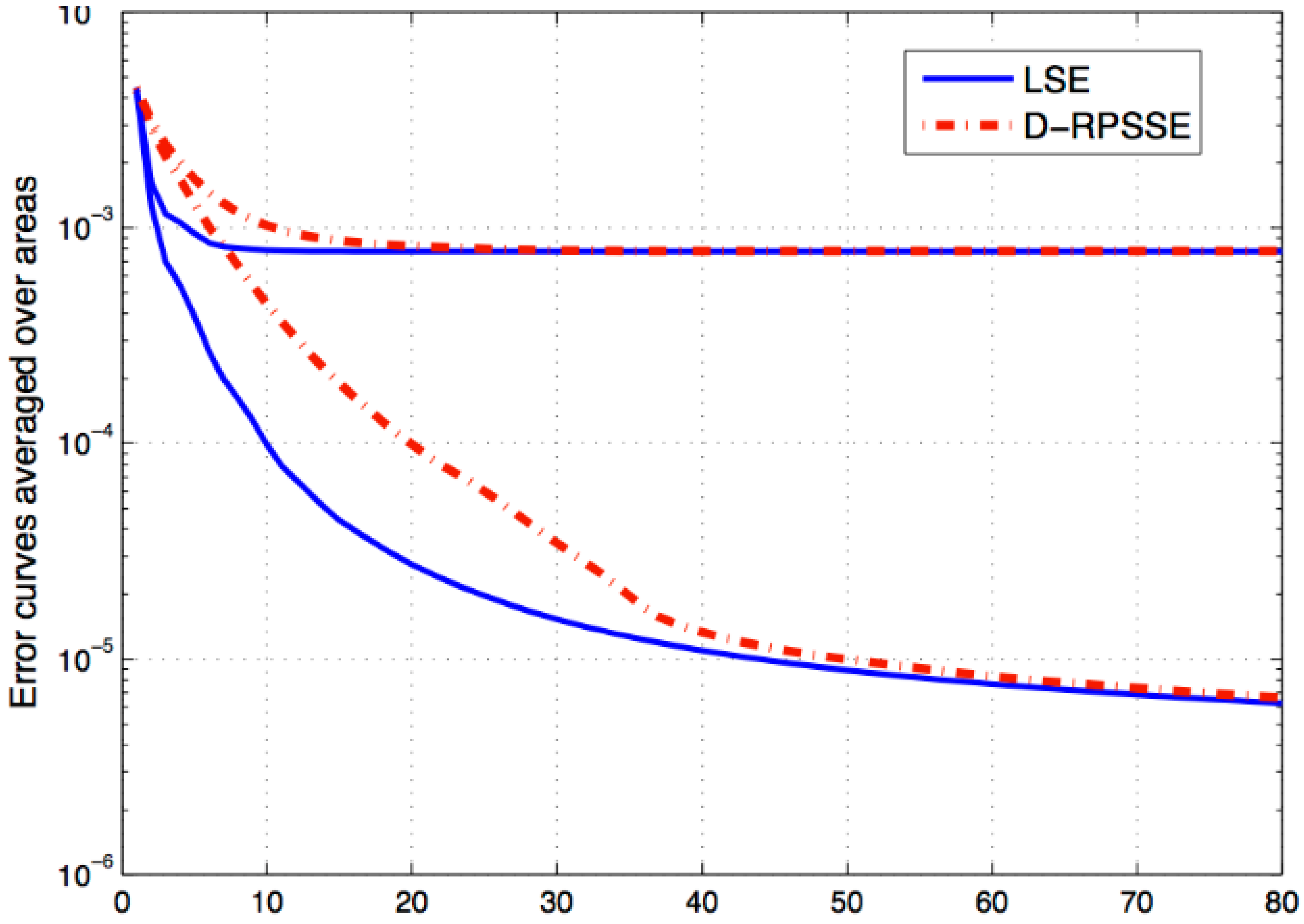
S1. $\mathbf{x}_k^{t+1} \Leftarrow \left(\mathbf{H}_k^T\mathbf{H}_k + c\cdot\mathbf{D}_k\right)^{-1}\left(\mathbf{H}_k^T(\mathbf{z}_k - \mathbf{o}_k^t) + c\cdot\mathbf{D}_k\mathbf{p}_k^t\right)$

S2. $\mathbf{o}_k^{t+1} \Leftarrow \left[\mathbf{z}_k - \mathbf{H}_k\mathbf{x}_k^{t+1}\right]_\lambda^+$



S3. $p_k^{t+1}(i) \Leftarrow p_k^t(i) + \left(x_l^{t+1}[i] - \frac{x_k^t(i) + x_l^t[i]}{2}\right)$

D-PSSE on a 4,200-bus grid

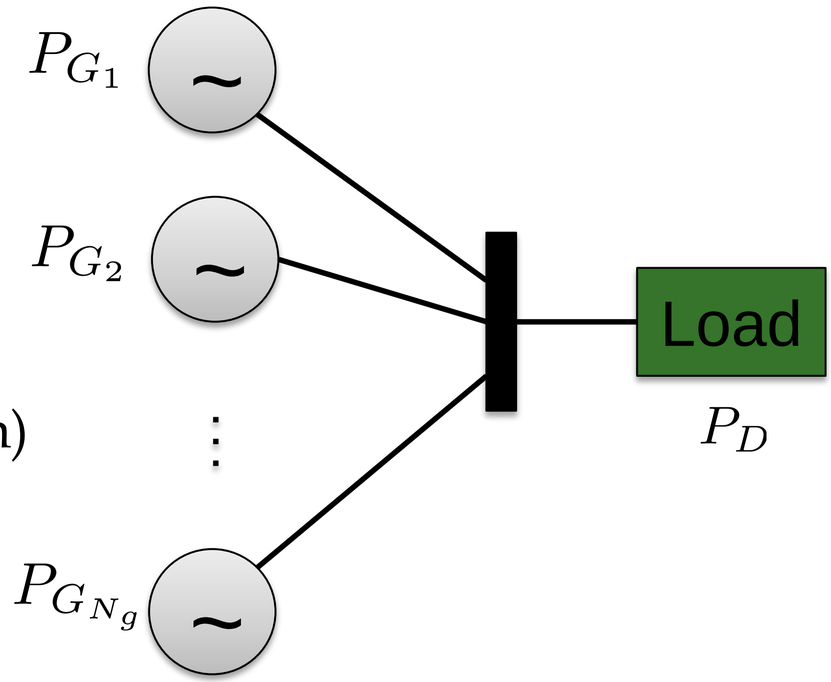


Optimal Power Flow

A. Gomez-Exposito, A. J. Conejo, and C. Canizares, *Electric Energy Systems: Analysis and Operation*, CRC, 2009.

A. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, 2nd ed., Wiley, 1996.

Generation cost



- Thermal generators
- Power output P_{G_i} (MW)
- Generation cost $C_i(P_{G_i})$ (\$/h or €/h)

Economic dispatch (ED): *Find most economically generated power output to serve given load*

- ED typically solved every 5-10 minutes

AC optimal power flow

- **Motivation**: Minimize generation cost respecting physical constraints

$$\min_{\mathbf{p}_G,\mathbf{q}_G,\mathbf{v}} \quad \sum_{m=1}^{N_b} C_m(P_{G_m})$$

$$\text{subj. to} \quad \mathbf{p}_G - \mathbf{p}_D + j(\mathbf{q}_G - \mathbf{q}_D) = \text{diag}(\mathbf{v})(\mathbf{Y}\mathbf{v})^*$$

$$|\text{Re}\{\mathcal{S}_{mn}\}| \leq f_{mn}^{\max}; \ |\mathcal{S}_{mn}| \leq S_{mn}^{\max}$$

$$V_m^{\min} \leq |\mathcal{V}_m| \leq V_m^{\max}$$

$$\mathbf{p}_G^{\min} \leq \mathbf{p}_G \leq \mathbf{p}_G^{\max}; \ \mathbf{q}_G^{\min} \leq \mathbf{q}_G \leq \mathbf{q}_G^{\max}$$

- Quadratic equality constraints ⇒ nonconvex problem
- Traditional approaches rely on KKT conditions

SDP relaxation

$$\mathcal{V}_m \mathcal{I}_m^* = \mathbf{e}_m^H \mathbf{v}(\mathbf{Y}\mathbf{v})^H \mathbf{e}_m = \mathrm{tr}[\mathbf{e}_m^H \mathbf{v}\mathbf{v}^H \mathbf{Y}^H \mathbf{e}_m] = \mathrm{tr}[\mathbf{Y}^H \mathbf{e}_m \mathbf{e}_m^H \mathbf{v}\mathbf{v}^H]$$

- Nodal balance constraint linear in $\mathbf{V} := \mathbf{v}\mathbf{v}^H$

$$P_{G_m} - P_{D_m} + j(Q_{G_m} - Q_{D_m}) = \mathrm{tr}[\mathbf{Y}^H \mathbf{e}_m \mathbf{e}_m^H \mathbf{V}]$$

- Line flow and bus voltage constrains also linear in $\mathbf{V}$
- AC-OPF with variables $\mathbf{p}_G, \mathbf{q}_G, \mathbf{V}$ and additional constraints

$\mathbf{V} \succeq \mathbf{0}$ $\mathrm{rank}[\mathbf{V}] = 1$ ⇒ Nonconvex ⇒ Drop

- Works in many practical OPF instances and IEEE benchmarks
- Optimal in tree graphs [Lam etal'12]

X. Bai, H.Wei, K. Fujisawa, and Y.Wang, "SDP for optimal power flow problems," Int. *J. El. Power-Ener. Syst.*, 2008.

J. Lavaei and S. Low, "Zero duality gap in optimal power flow problem," *IEEE Trans. Power Syst.*, Feb. 2012.

AC OPF for multi-phase

- Power and voltage magnitude as linear functions of $\mathbf{V} := \mathbf{v}\mathbf{v}^H$
- Regulating constraints per node *and* per phase (can be unbalanced)

$$\min_{\mathbf{V},\mathbf{p}_G,\mathbf{q}_G} \quad C(\mathbf{p}_G, \mathbf{q}_G)$$

$$\text{sub. to } \{P_{G,n}^{\phi}, Q_{G,n}^{\phi}\} \in \mathcal{B}_n^{\phi}, \text{ and } \forall \phi, n$$

$$\mathrm{Tr}(\mathbf{\Phi}_n^{\phi}\mathbf{V}) = P_{G,n}^{\phi} - P_{\ell,n}^{\phi}$$

$$\mathrm{Tr}(\mathbf{\Psi}_n^{\phi}\mathbf{V}) = Q_{G,n}^{\phi} - Q_{\ell,n}^{\phi}$$

$$(V^{\min})^2 \leq \mathrm{Tr}(\mathbf{M}_n^{\phi}\mathbf{V}) \leq (V^{\max})^2$$

$$\mathbf{V} \succeq \mathbf{0}, \; \cancel{\mathrm{rank}(\mathbf{V}) = 1}$$

- $\mathrm{rank}(\mathbf{V}^{\mathrm{opt}}) = 1$ ➡ globally optimal AC OPF solution!

E. Dall'Anese, H. Zhu, and G. B. Giannakis, "Distributed optimal power flow for smart microgrids," *IEEE Trans. Smart Grid*, vol. 4, no. 3, pp. 1464-1475, Sep. 2013.

Distributed three-phase OPF

- Multi-area based on non-convex OPF
  [Kim-Baldick'97, Hug-Andersson'09, Erseghe'14]

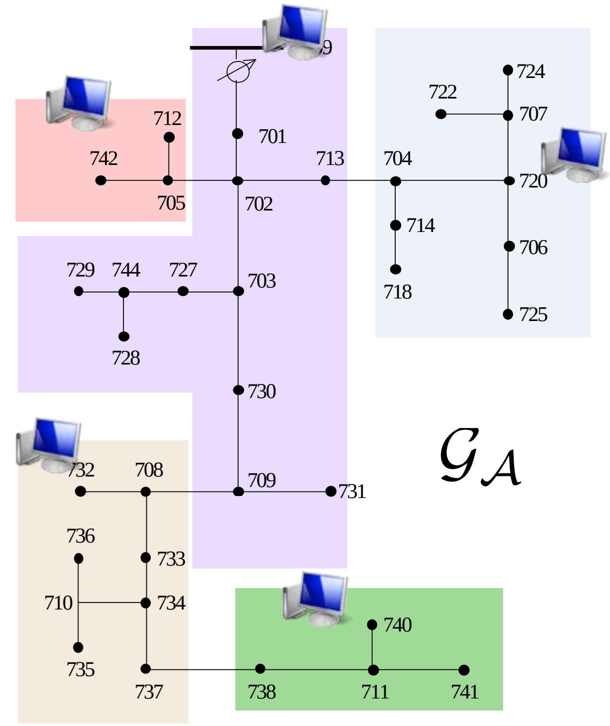
- Node-to-node for single-phase systems
  [Zhang et al'12]

- Distributed SDP for three-phase systems

$$\min_{\mathbf{V},\{\mathbf{s}_n\}} \sum_a C_a(\mathbf{V}^{(a)}, \mathbf{s}^{(a)})$$

$$\text{sub. to} \quad \{\mathbf{V}^{(a)}, \mathbf{s}^{(a)}\} \in \mathcal{B}^{(a)} \ \forall\, a$$

$$\mathbf{V} \succeq \mathbf{0}$$



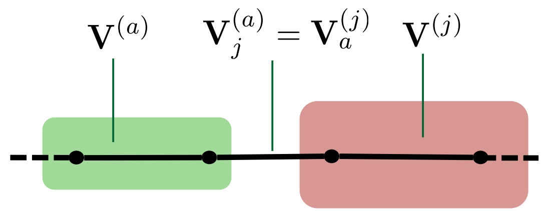
Challenge: PSD constraint couples local quantities! **Q**: $\mathbf{V} \succeq \mathbf{0} \stackrel{?}{\longleftrightarrow} \{\mathbf{V}^{(a)} \succeq \mathbf{0}\}$

Topology-based decoupling

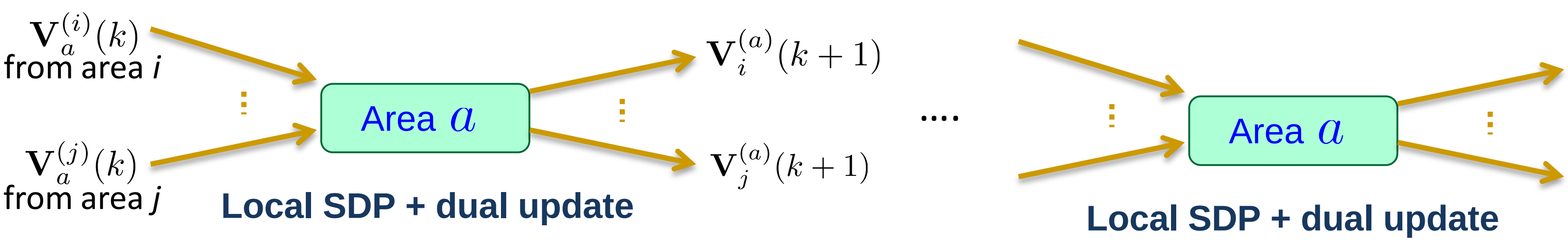
Result: If the *graph of areas* is a tree, without "loops" across areas,

then the centralized PSD constraint *decouples*

$$
\begin{aligned}
\{\mathbf{V}^{(a)}_{\text{opt}}, \mathbf{s}^{(a)}_{\text{opt}}\} = \arg \min_{\{\mathbf{V}^{(a)}\},\{\mathbf{s}^{(a)}\}} \; & \sum_a C_a(\mathbf{V}^{(a)}, \mathbf{s}^{(a)}) \\
\text{sub. to} \; & \{\mathbf{V}^{(a)}, \mathbf{s}^{(a)}\} \in \mathcal{B}^{(a)} \;\; \forall \, a \\
& \mathbf{V}^{(a)} \succeq \mathbf{0} \quad \forall \, a \\
& \mathbf{V}^{(a)}_j = \mathbf{V}^{(j)}_a \quad \forall \, (a, j) \in \mathcal{E}_{\mathcal{A}}
\end{aligned}
$$

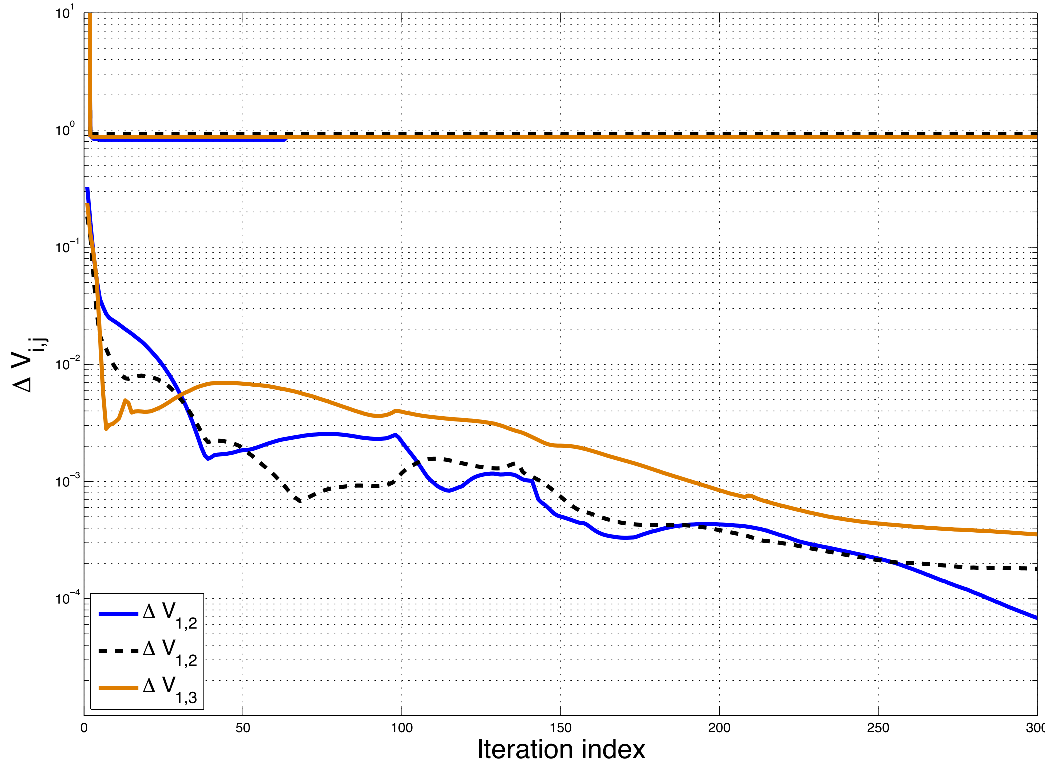


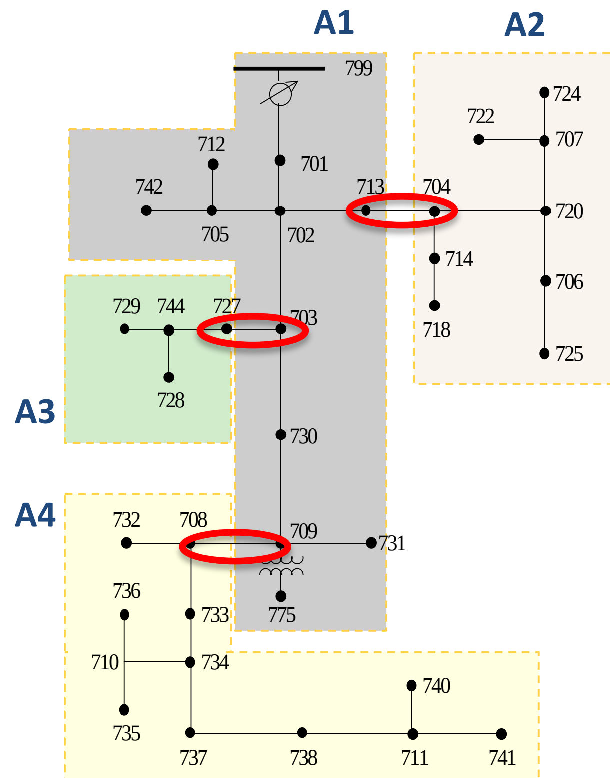
- Distributed solution via alternating direction method of multipliers



Illustrative test case

- Consensus error





- Comparison with sub-gradient [Zhang et al'12]
- Convergence rate does not depend on area size

Demand Response

Motivation for DR

- Changes in electricity consumption by end-users in response to
  - Changes in electricity prices over time
  - Incentive payments at times of high wholesale prices

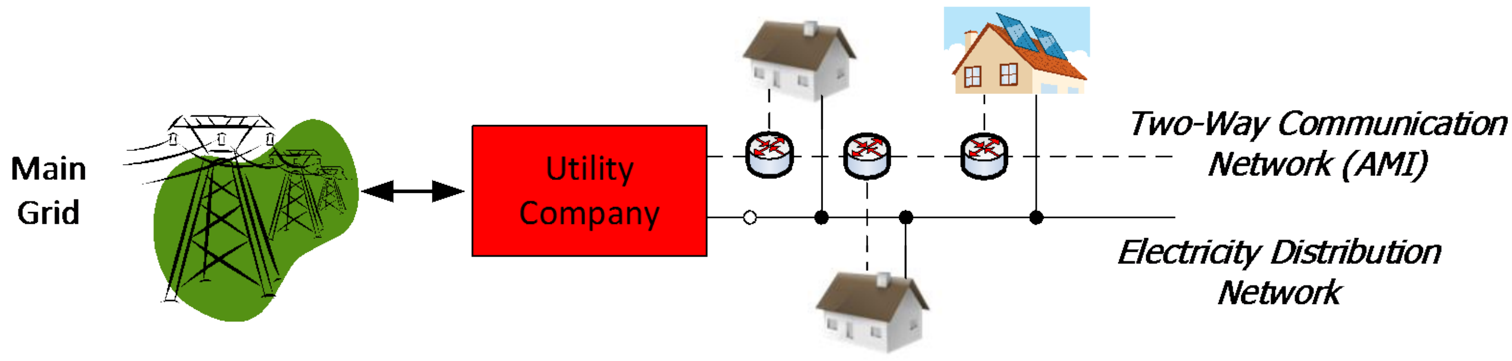
- Benefits of DR
  - Reduced demand reduces the potential of forced outages
  - Lower demand holds down electricity prices in spot markets
  - Can reduce the amount of generation and transmission assets

- DR programs
  - Incentive-based programs
  - Price-driven programs

US DoE, "Benefits of DR in Electricity Markets and Recommendations for Achieving Them," Feb. 2006
K. Hamilton and N. Gulhar, "Taking demand response to the next level," *IEEE PES Mag.*, May 2010.

Cooperative DR

- Set of users (residences) $\{1, \ldots, R\}$ served by the same utility
- Set of smart appliances $\mathcal{A}_r$ per user r
- Power consumption p_{ra}^t
- End-user utility function $U_{ra}(\mathbf{p}_{ra})$
- Cost of power procurement for utility company $C^t\left(\sum_{r=1}^{R}\sum_{a\in\mathcal{A}_r} p_{ra}^t\right)$



Social welfare maximization

$$\min_{\{\mathbf{p}_{ra}\}} \quad \sum_{t=1}^{T} C^t \left(\sum_{r=1}^{R} \sum_{a \in \mathcal{A}_r} p_{ra}^t \right) - \sum_{r=1}^{R} \sum_{a \in \mathcal{A}_r} U_{ra}(\mathbf{p}_{ra})$$

$$\text{subj. to} \quad \mathbf{p}_{ra} \in \mathcal{P}_{ra},\ a \in \mathcal{A}_r,\ r = 1 \ldots, R$$

- **Motivation:** Reduce peak demand respecting preferences of users

- Convexity depends on $\mathcal{P}_{ra}$
- Challenges
  - Scalable scheduling over AMI; and privacy issues

Solution approaches

- Gradient projection, block coordinate descent, dual decomposition

  [Chen etal'12], [Mohsenian-Rad etal'10], [Papavasiliou etal'10], [Samadi etal'11], [Gatsis-GG'12]

- **Dual decomposition:** Introduce variable s^t for total supplied power

$$\sum_{r=1}^{R} \sum_{a \in \mathcal{A}_r} p_{ra}^t \leq s^t$$

Demand-supply balance

  - Lagrange multiplier λ^t for supply-demand balance

- **Upshot**
  - Sub-problems for utility and smart meters are separated
  - Privacy respected

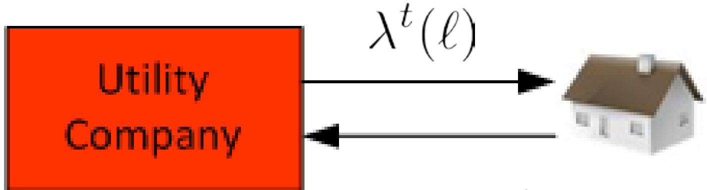
Distributed DR algorithm

- **Schedule update:** At the utility company and smart meters

$$\min_{0 \leq s^t \leq s^{\max}} \{C^t(s^t) - \lambda^t(\ell) s^t\}$$

prices

$\lambda^t(\ell)$



$$\min_{\mathbf{p}_{ra} \in \mathcal{P}_{ra}} \sum_{t=1}^{T} \{\lambda^t(\ell) p_{ra}^t - U_{ra}(\mathbf{p}_{ra})\}$$

$\sum_{a \in \mathcal{A}} p_{ra}^t(\ell)$

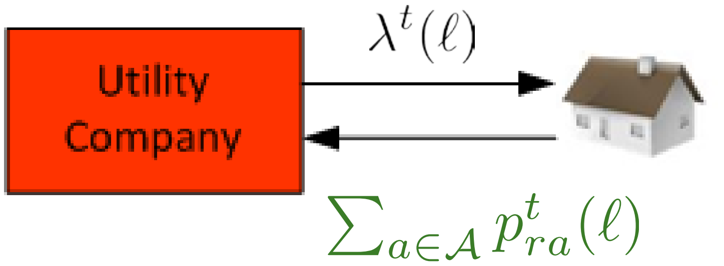
total hourly consumption

- **Multiplier update:** At utility company

$$\lambda^t(\ell + 1) = \left[\lambda^t(\ell) + \beta\left(\sum_{r=1}^{R} \sum_{a \in \mathcal{A}} p_{ra}^t(\ell) - s^t(\ell)\right)\right]^+$$

Lost AMI messages

- Messages in both ways may be lost
  - Not transmitted, due to failure
  - Not received, due to noise
  - Cyber-attacks



- Use the latest message available
- Convergence established for different lost-message patterns
  - Asynchronous subgradient method

- **Benefit:** Resilience to communication network outages

N. Gatsis and G. B. Giannakis, "Residential load control: Distributed scheduling and convergence with lost AMI messages," *IEEE Trans. Smart Grid*, vol. 3, pp. 770-786, June 2012.

Plug-in Electric Vehicles

Plug-in electric vehicles

- PEVs feature batteries that can be plugged in
  - At end-user premises
  - At charging stations

- Benefits of high PEV penetration
  - Environmental: reduce carbon emissions
  - Economic: reduce dependency on oil

- Charging coordination is well motivated to avoid
  - Overloading of distribution networks [Clement-Nyns et al'10]
  - Creating new peaks

Charging coordination

- Fleet of vehicles $n = 1, \ldots, N$ to charge on top of baseload D^t
- Fraction of charge (rate) r_n^t per slot $t = 1, \ldots, T$
  - Vehicle plugged in at different slots ⇨ $0 \leq r_n^t \leq r_n^{t,max}$

- Centralized charging coordination

$$\min_{\{\mathbf{r}_n\}} \sum_{t=1}^{T} C^t \left(D^t + \sum_{n=1}^{N} r_n^t \right)$$

$$\text{subj.to} \quad \left. \begin{array}{l} 0 \leq r_n^t \leq r_n^{t,max} \\ \sum_{t=1}^{T} r_n^t = R_n \end{array} \right\} \mathcal{E}_n \quad n = 1, \ldots, N$$

  - Convex and differentiable $C^t(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$, e.g., $C^t(\cdot) := (\cdot)^2$

L. Gan, U. Topcu, and S. Low, "Optimal decentralized protocol for electric vehicle charging," *Proc. Conf. on Decision and Control*, 2011.

Distributed PEV scheduling

- Relies on Frank-Wolfe (FW) optimization method
  - Identical per-vehicle partial gradients of the costs

$$\nabla_{\mathbf{r}_n}\left(\sum_{t=1}^{T} C^t\left(D^t + \sum_{n=1}^{N} r_n^t\right)\right) = \mathbf{g},\ \ n = 1, \ldots, N$$

  - At iteration i, vehicle n solves a linear program

$$\hat{\mathbf{r}}_n(i) \in \arg\min_{\hat{\mathbf{r}}_n \in \mathcal{E}_n} \hat{\mathbf{r}}_n^\top \mathbf{g}(i)$$

$\hat{\mathbf{r}}_n(i)$ ← auxiliary variable

Solution: charge first small entries of $\mathbf{g}(i)$

  - Vehicle n updates $\mathbf{r}_n(i+1) = (1 - \eta(i))\,\mathbf{r}_n(i) + \eta(i)\hat{\mathbf{r}}_n(i)$

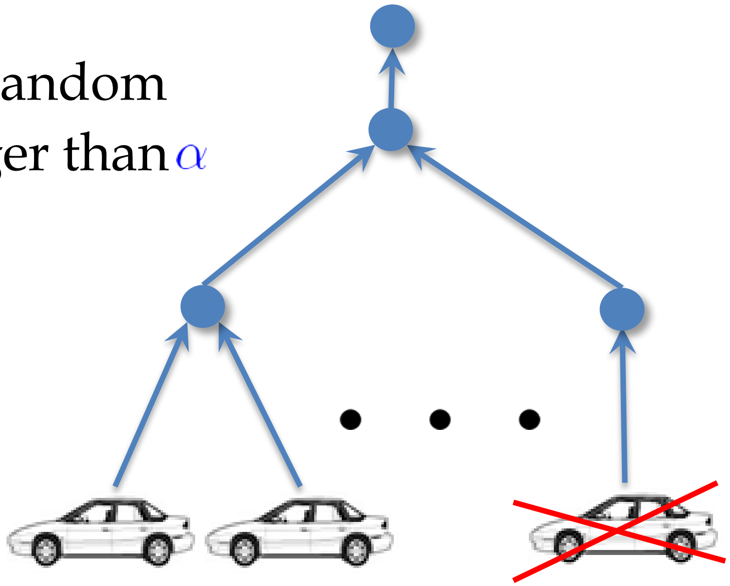
$\eta(i) := \frac{2}{i+2}$ or chosen via line-search

L. Zhang, V. Kekatos, and G. B. Giannakis, "Scalable electric vehicle charging protocols," *IEEE Trans. Power Syst.*, 2016 (to appear).

Asynchronous updates

- To deal with *process delays* of EV controllers and/or *commutation failures*

(as1) Lost updates occur independently at random
(as2) Probability of a successful update larger than α



- Guaranteed convergence with

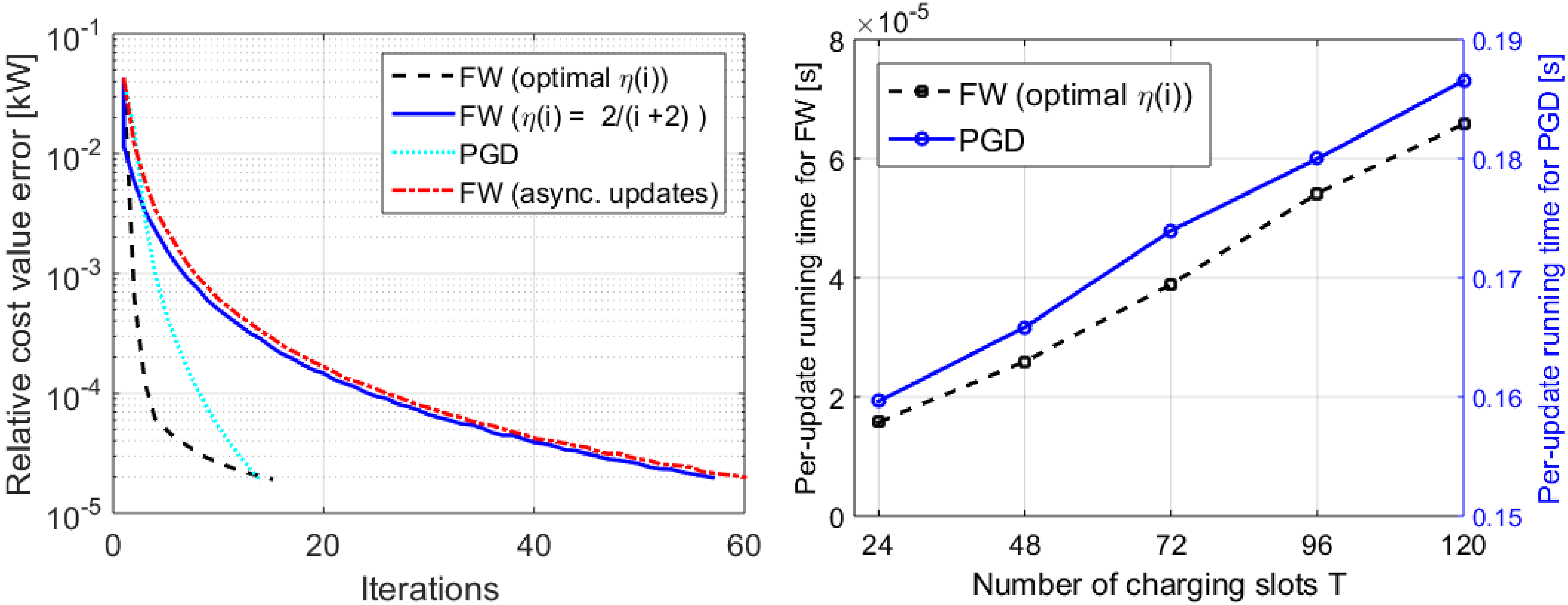
$$\eta(i) := \frac{2}{\alpha i+2}$$

  - $\mathcal{O}(1/i)$ convergence rate in expectation

L. Zhang, V. Kekatos, and G. B. Giannakis, "A generalized Frank-Wolfe approach to decentralized control of vehicle charging," *Proc. Conf. on Decision and Control*, 2016.

Numerical tests

- 51 out of 52 EVs are updated in an asynchronous setting



- Projected gradient descent (GD) and ADMM must project (expensive!)
- *Speed-up advantage* of FW thanks to simple updates

Take-home messages

- Distributed and robust PSSE
  - Non-convexity tackled via semidefinite relaxation
  - Decentralized estimation via *ADMM*
  - Sparse outlier models for robustness to ``bad data''
- Distributed OPF
  - Semidefinite relaxation is tight for radial microgrids
  - *ADMM* solver for decentralized multiphase OPF
- Distributed DR
  - Decentralized management through *dual decomposition*
  - Resilience to lost AMI messages
- Distributed EV charging
  - Scalable and decentralized scheduler via *Frank-Wolfe* iteration
  - Robust to random communication outages

Thank you!