A Robust Event-Triggered Consensus Strategy for Linear Multi-Agent Systems with Uncertain Network Topology

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Multi-agent Systems

A multi-agent system consists of multiple agents that interact to achieve a cooperative objective.

An agent can represent a moving vehicle, a sensor node, an electric bus, etc.

Cooperative Objectives: Formation, Consensus, Containment, Rendezvous, ...
Consensus Control:

Agent 1
Agent 2
Agent 3
Agent 4

★ The agreement value

**Consensus**: To reach an agreement upon a common value.
Agent Dynamics

General Linear Agent Model:

\[ \dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad 1 \leq i \leq N \]  

1. \( x_i(t) \in \mathbb{R}^n \): The state of agent \( i \) at time instant \( t \);
2. \( A \in \mathbb{R}^{n \times n} \): System matrix (known and constant);
3. \( B \in \mathbb{R}^{n \times m} \): Input matrix (known and constant);
4. \( u_i(t) \in \mathbb{R}^{m \times n} \): A proposed distributed control input;
5. \( N \): Number of agents in the network.
Consensus Definition:

For any initial condition $x_i(0)$, the consensus problem for (1) is said to be solved iff:

- **Global sense:** $\lim_{t \to \infty} \| x_i(t) - x_j(t) \| = 0, \ (1 \leq i, j \leq N)$,

- **Average sense:** $\lim_{t \to \infty} \| x_i(t) - \frac{1}{N} \sum_{j=1}^{N} x_j(0) \| = 0, \ (1 \leq i \leq N)$,

Average consensus is usually considered for first-order agents defined by $\dot{x}_i(t) = u_i(t)$, with $x_i(0)$ as initial local observation.

Key components in reaching consensus:

- Distributed control input $u_i(t)$,
- Information exchange between the neighbouring agents.
Event-triggered Consensus

\[ \hat{x}_j(t), \quad j \in N_i \]

1. \( x_i(t) \): The state of agent \( i \)
2. \( \hat{x}_i(t) \): The last transmitted state of agent \( i \) up to time \( t \)

The received information is subject to uncertainty due to existence of communication unreliabilities → robustness is required
Motivation:

- Transmission saving for consensus in multi-agent systems with bandwidth constrained environments and unreliable channel.

Objective:

- Achieve event-triggered consensus with a desired exponential rate of convergence (as opposed to asymptotic rate);
- Compute optimal consensus parameters to achieve consensus in presence of network uncertainties.
Main Features

- **Event-based disagreement vector**: 
  \[ q_i(t) = \sum_{j \in \mathcal{N}_i} \bar{a}_{ij} \left( e^{A(t-t_{k_i}^i)} x_i(t_{k_i}^i) - e^{A(t-t_{k_j}^i)} x_j(t_{k_j}^i) \right) \]
  where \( \bar{a}_{ij} \) is the uncertain (but norm-bounded) weight for channel link between agent \( i \) and \( j \).

- **Measurement error**: 
  \[ e_i(t) = e^{A(t-t_{k_i}^i)} x_i(t_{k_i}^i) - x_i(t) \].

- **Event-triggering function**: given an event time \( t_{k_i}^i \), the next event for agent \( i \) is triggered at \( t = t_{k_i}^i + 1 \), where
  \[ t_{k_i}^i + 1 = \inf \{ t > t_{k_i}^i \mid \| e_i(t) \| - \phi \| q_i(t) \| \geq 0 \} \],
  (2)
  \( \phi > 0 \) : Transmission threshold to be designed.
The proposed control law:

\[ u_i(t) = K_i q_i(t), \]  

(3)

\( K_i \): Control gain to be designed.

\textbf{Question}: How to design optimal\(^1\) values for transmission threshold \( \phi \) and control gain \( K_i \) that guarantee an exponential rate of consensus in \textit{norm-bounded} uncertain network channel?

\(^1\)maximize \( \phi \) to minimize events, and minimize \( K_i \) to minimize control force
Preliminary steps prior to optimization

- Consider the augmented closed-loop system;

- Convert the consensus problem into an equivalent stability problem $\rightarrow$ Lyapunov stability method

- Obtain sufficient conditions and inequalities for uncertain connectivity links.
Compute optimal consensus parameters

Solve the following convex optimization problem with desired convergence rate $\zeta$

$$\min_{\Theta_i, \mu, \epsilon, \tau_j, P, \omega_1, \omega_2, \omega_3, \omega_4} \quad f = \omega_1 + \omega_2 + \omega_3 + \omega_4$$

S.t:

$$\Pi = \begin{bmatrix} \Pi_1 & \Pi_2 \\ * & \Pi_3 \end{bmatrix} < 0, \quad \begin{bmatrix} -\omega_1 & \tau_1 \\ * & -1 \end{bmatrix} < 0, \quad \begin{bmatrix} -\omega_2 & \mu \\ * & -1 \end{bmatrix} < 0,$$

$$\begin{bmatrix} \omega_3 I & I \\ * & P \end{bmatrix} > 0, \quad \begin{bmatrix} -\omega_4 I & \Theta^T \\ * & -I \end{bmatrix} < 0,$$

- $\Theta_i \ (1 \leq i \leq N)$, $\mu$, $\epsilon$, $\tau_j \ (1 \leq j \leq 3)$, $P$, $\omega_c \ (1 \leq c \leq 4)$ are decision variables;
- Block Matrix $\Pi$ contains information about agent models, network connectivity, exponential convergence criterion, uncertainty upper bound, control gain $K_i$, and transmission threshold $\phi$. 
Compute optimal consensus parameters

Once the optimization problem (4) is solved, compute consensus parameters

$$\phi = \sqrt{\tau_1^{-1}\mu^{-1}}, \text{ and } K_i = B_i^\dagger P^{-1} \Theta_i, \quad (1 \leq i \leq N)$$ \hspace{1cm} (5)

Consensus parameters are bounded for the minimized objective function $f = \omega_1 + \omega_2 + \omega_3 + \omega_4$

$$\phi \geq (\omega_1\omega_2)^{-\frac{1}{4}}, \quad K_i^T K_i \leq \omega_3 \omega_4^2 B_i^\dagger B_i^{\dagger T}, \quad (1 \leq i \leq N).$$ \hspace{1cm} (6)
Experimental Results

- A network of six second-order heterogeneous agents

\[ \dot{r}_i(t) = v_i(t), \]
\[ m_i \ddot{v}_i(t) = u_i(t), \quad (1 \leq i \leq 6), \quad (7) \]

\[ r_i(t) \in \mathbb{R}: \text{Position,} \quad v_i(t) \in \mathbb{R}: \text{Velocity,} \quad m_i: \text{Inertia} \]

- Consensus in this problem is to, distributively, reach a common position and velocity

- Laplacian Matrix (two unreliable links)

\[ L = \begin{bmatrix}
2.5 & 0 & 0 & -0.5 & -1 & -1 \\
0 & 2 & 0 & 0 & -1 & -1 \\
0 & -1 & 3 & -1 & 0 & -1 \\
0 & -1 & 0 & 2 & 0 & -1 \\
-1 & -1 & 0 & -1 & 3 & 0 \\
-1 & -1 & -0.5 & 0 & 0 & 2.5
\end{bmatrix} \quad \longleftrightarrow \quad \bar{L} = \begin{bmatrix}
2 & 0 & 0 & 0 & -1 & -1 \\
0 & 2 & 0 & 0 & -1 & -1 \\
0 & -1 & 3 & -1 & 0 & -1 \\
0 & -1 & 0 & 2 & 0 & -1 \\
-1 & -1 & 0 & -1 & 3 & 0 \\
-1 & -1 & 0 & 0 & 0 & 2
\end{bmatrix} \quad (8) \]

- Solve the optimization problem (4) to compute \( K_i \) and \( \phi \)
How different values for decay rate $\zeta$ affect the consensus process?

<table>
<thead>
<tr>
<th>decay rate $\zeta$</th>
<th>Number of transmissions per agent</th>
<th>Consensus time (sec)</th>
<th>Objective function $f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>262, 295, 333, 318, 369, 321</td>
<td>10.57</td>
<td>401.19</td>
</tr>
<tr>
<td>0.3</td>
<td>133, 154, 175, 180, 164, 142</td>
<td>4.81</td>
<td>406.84</td>
</tr>
<tr>
<td>0.4</td>
<td>68, 58, 95, 194, 50, 68</td>
<td>3.51</td>
<td>411.27</td>
</tr>
</tbody>
</table>

Table 1: Consensus performance for varying $\zeta$. 

Figures (a), (b), (c), and (d) illustrate the position and velocity for agent $i$ and agent 4, respectively.
Conclusion

1. For a desired rate of convergence, robust event-triggered consensus is reached for norm-bounded uncertain networks;

2. Using convex optimization, the transmission threshold $\phi$ is maximized (to trigger minimum number of events) and control gain $K_i$ is minimized (to minimize the control force);

3. As convergence rate $\zeta$ is increased, the consensus time constantly gets reduced until the optimization problem becomes infeasible.
Thank You