An Event-triggered Average Consensus Algorithm with Performance Guarantees for Distributed Sensor Networks

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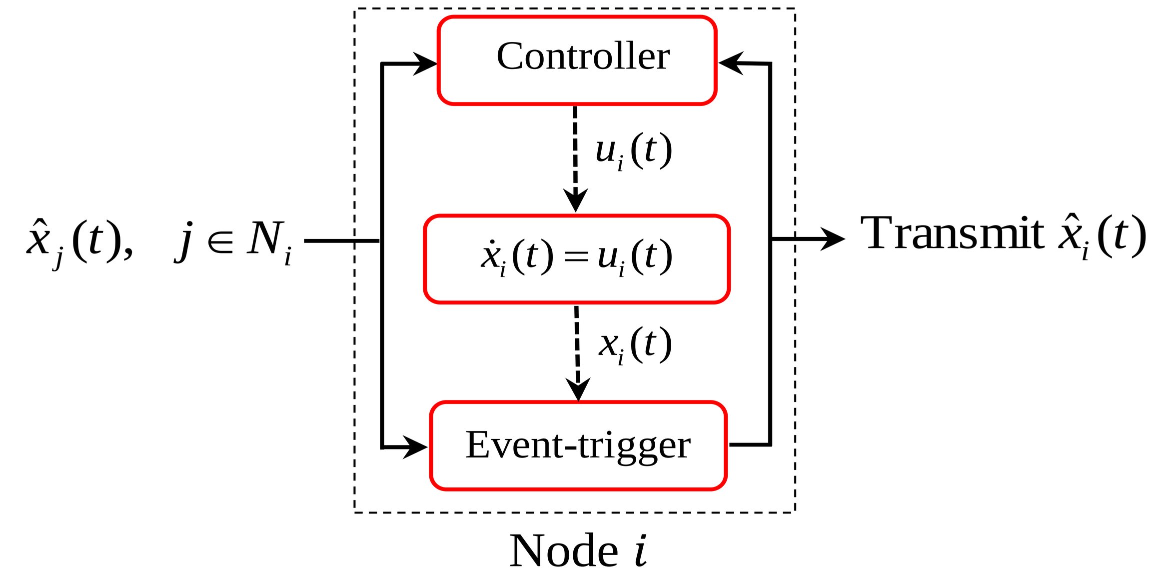
Motivation



Motivation:

- Energy and communication savings for distributed sensor networks operating in bandwidth constrained environments.
- Applying event-triggered framework to average consensus in multi-agent networks.

Objective:

- Formulate the problem as a guaranteed cost optimization and compute the design parameters for the event-triggered average consensus framework.

Problem Statement

Consensus framework: $\dot{x}_i(t) = \underbrace{u_i(t)}_{\text{input}}, \quad 1 \le i \le N$

Average consensus is achieved if and only if $\lim_{t\to\infty} \left| x_i(t) - 1/N \sum_{j=1}^{N} x_j(0) \right| = 0, \quad (1 \le i, j \le N)$.

Let $\hat{x}_i(t) = x(t_k^i)$, where t_k^i is the time instant for the most recent event for agent i.

Event-based Control Input:

$$u_i(t) = -k \sum_{j \in \mathcal{N}_i} \big(\hat{x}_i(t) - \hat{x}_j(t)\big), \quad (1 \le i \le N), \qquad (1)$$

- k: scalar control gain to be computed.

Event-triggering function: Given t_k^i, the next event for agent i is triggered at $t = t_{k+1}^i$ where t_{k+1}^i satisfies

$$t_{k+1}^i = \inf\{\, t > t_k^i : |e_i(t)| - \phi_i |\hat{\mathbb{X}}_i(t)| \ge 0 \}, \qquad (2)$$

- $e_i(t) = x_i(t) - \hat{x}_i(t)$
- $\hat{\mathbb{X}}_i(t) = \sum_{j \in \mathcal{N}_i} \big(\hat{x}_i(t) - \hat{x}_j(t)\big)$,
- ϕ_i: transmission threshold to be computed for agent i

How to compute k and ϕ_i?

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Parameter Optimization

- Consider the cost function

$$J = \int_0^\infty \boldsymbol{x}^T(t) R \boldsymbol{x}(t) + \boldsymbol{u}^T(t) Q \boldsymbol{u}(t) dt \qquad (3)$$

- R: desired weight to the state $\boldsymbol{x}(t)$ to control the consensus convergence rate;
- Q: desired weight to the difference vector $\boldsymbol{e}(t)$ to control the number of transmission events.

Design k and ϕ_i in such a way that the associated cost J with the event-triggered average consensus satisfies $J \le J^*$, where J^* is said to be the guaranteed cost

$$\min_{\mu,\tau,p,\gamma_i,\omega_\tau,\omega_\mu,\omega_p,\omega_{\gamma_i}} f = \omega_\tau + \omega_k + \omega_p + p + \mathrm{Tr}(\boldsymbol{\omega}_\Gamma) \qquad (4)$$

$$\Pi \triangleq \begin{bmatrix} -\mu\mathbb{L} - \mu\mathbb{L}^T + R & -\mu\mathbb{L} & M^T \\ * & -\tau I + Q & M^T \\ * & * & -\Gamma \end{bmatrix} < 0, \begin{bmatrix} \omega_p & 1 \\ * & p \end{bmatrix} > 0,$$

$$\begin{bmatrix} -\boldsymbol{\omega}_\Gamma & \Gamma \\ * & -I \end{bmatrix} < 0, \begin{bmatrix} -\omega_\tau & \tau \\ * & -1 \end{bmatrix} < 0, \begin{bmatrix} -\omega_\mu & \mu \\ * & -1 \end{bmatrix} < 0,$$

- $\mu, \tau, p, \gamma_i, \omega_\tau, \omega_\mu, \omega_p, \omega_{\gamma_i}$ are optimization decision variables;
- R and Q are given matrix weights;
- M, and $\mathbb{L}$ contain network connectivity information;
- Compute k and ϕ_i as follows

$$k = p^{-1}\mu, \quad \text{and} \quad \phi_i = \sqrt{\tau^{-1}\gamma_i^{-1}}, \quad (1 \le i \le N), \qquad (5)$$

- Using (5), it holds that $J \le \{J^* = x^T(0) L^T p L x(0)\}$

The Consensus Algorithm

Algorithm 1. The GP-ETAC Algorithm

Input: Adjacency Weighting Matrix $\mathcal{A} = \{a_{ij}\}$, Initial conditions $x_i(0)$, and Weighting Matrices $\{R, Q\}$.

Output: Event-triggered Average Consensus with Guaranteed Performance

Preliminaries: (P1 – P2)

P1. Remove the N^{th} row of L to determine $\hat{L}$ and $\mathbb{L} = \hat{L} L \hat{L}^\dagger$.

P2. Given $\hat{L}$, determine $\boldsymbol{\alpha}$ and matrix M from Lemma 2.

Optimization and Parameter Design Steps: (D1–D2)

D1. Using a convex optimization solver, solve the minimization problem (14) for given parameters $\{R, Q\}$.

D2. Using (13), compute transmission threshold ϕ_i $(1 \le i \le N)$ and control gain k.

Consensus Steps: (C1 – C4)

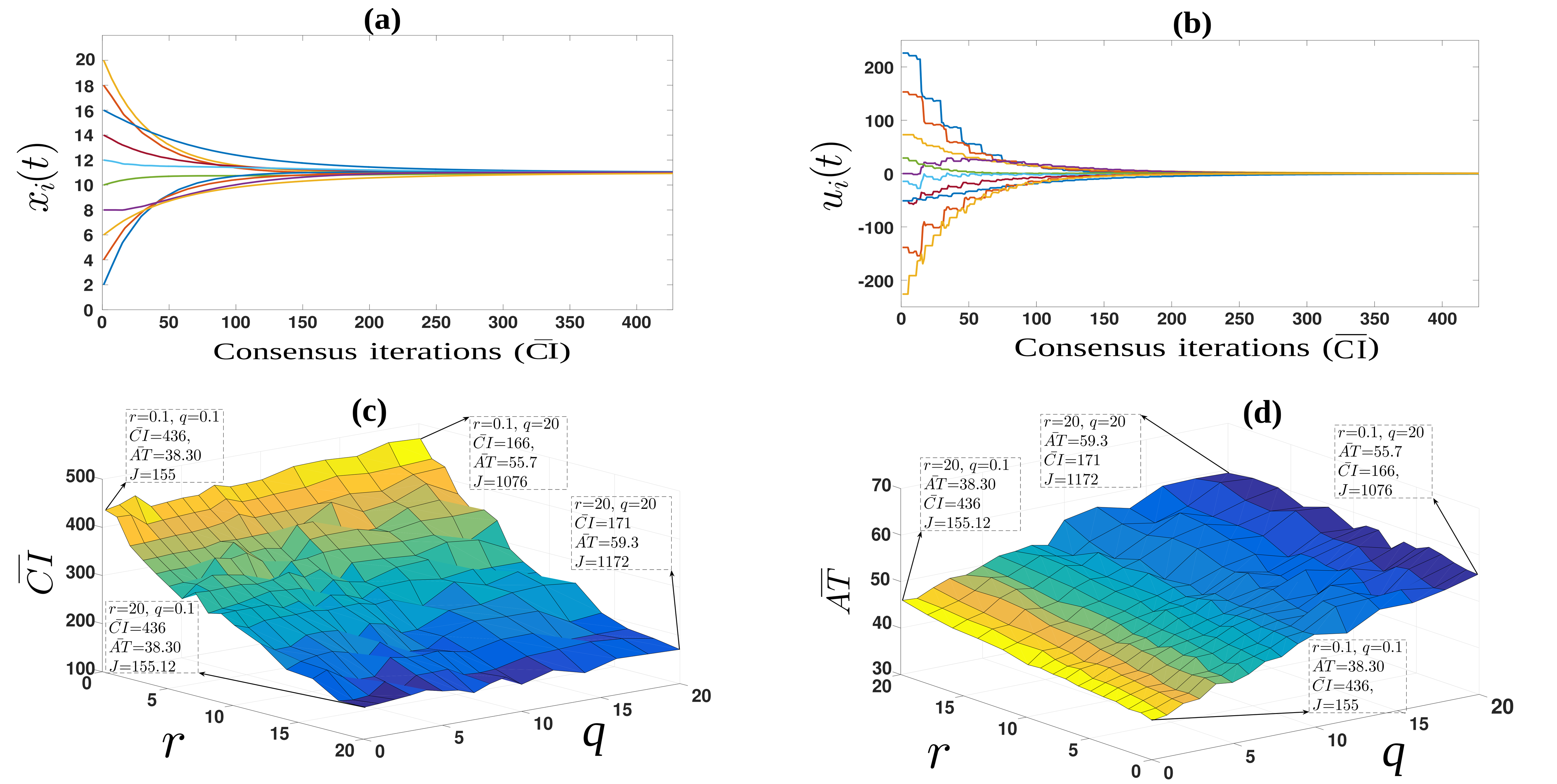
C1. Each sensor sends its initial value $x_i(0)$ to its neighbours.

C2. In each consensus iteration, the state of node i is excited by control law (3) with k computed from D2.

C3. In each consensus iteration, the event-triggering condition (8) is locally monitored with the designed ϕ_i to determine when to transmit $x_i(t)$ to the neighbours.

C4. Steps C2 and C3 continue until average consensus (i.e., $u_i(t) \to 0$ in (3)) is achieved among agents.

Monte-Carlo Simulations



One randomly selected simulation (Figs. (a) and (b)):

- Non-zero elements in the symmetric adjacency matrix A of a random network with $N = 10$: $\{a_{14}, a_{16}, a_{17}, a_{19}, a_{1,10}, a_{24}, a_{26}, a_{29}, a_{2,10}, a_{37}, a_{39}, a_{45}, a_{48}, a_{5,10}, a_{67}, a_{68}, a_{6,10}, a_{78}, a_{7,10}, a_{9,10}\}$.
- Initial optimization problem (4) with $R = rI$ and $Q = qI$ with $r = 1$ and $q = 1$
- Optimized parameters: k=3.939, ϕ_1=0.055, ϕ_2=0.065, ϕ_3=0.034, ϕ_4=0.035, ϕ_5=0.050, ϕ_6=0.046, ϕ_7=0.041, ϕ_8=0.032, ϕ_9=0.056, ϕ_{10}=0.020.
- 462 iterations ($\overline{\text{CI}}$) to reach average consensus.
- Number of events for each agent: 29, 26, 47, 44, 33, 34, 37, 47, 27, and 71 $\to$ Average transmission ($\overline{\text{AT}}$) value of 39.50 times per agent.
- Computed value of the guaranteed cost J^* is 824.76.
- Cost of consensus process is $J = 151.67 \to J \le J^*$

Effect of r and q on random networks (Figs. (c) and (d)):

- Investigate the effect of different choices of $\{r, q\}$ (R=rI and Q=qI) on the average consensus.
- With a fixed q, increasing r accelerates the convergence rate (smaller $\overline{\text{CI}}$) at the expense of higher $\overline{\text{AT}}$ and increased cost J (Table 1).
- Decreasing q for a fixed r leads to a smaller $\overline{\text{AT}}$ (larger gaps between triggering moments are allowed)

Table 1: Impact of weighting matrices r and q on GP-ETAC.

r	q	k	mean(ϕ_i)	$\overline{\text{CI}}$	$\overline{\text{AT}}$	J	J^*
4	1	5.0731	0.0404	333	43.90	450.15	794.71
8	1	6.0091	0.0370	282	47.50	756.35	858.43
1	20	3.8875	0.0373	434	45.70	272.53	795.11
1	0.1	3.9256	0.0441	428	38.80	144.01	849.31

Conclusion

- *Linear matrix inequality (LMI)* optimization computes *optimal* consensus parameters, i.e., a control gain and transmision thresholds, guaranteeing the *minimum cost* for the event-triggered average consensus process.
- To guarantee average consensus, the control gain and transmission thresholds are *coupled* to benefit from a *unified multi-objective optimization*.
- Based on desired weighting matrices R and Q, an *optimized trade-off* between consensus convergence rate and the number of transmissions is developed for event-triggered average consensus in distributed sensor networks.