Event-based Consensus for a Class of Heterogeneous Multi-agent Systems: An LMI Approach

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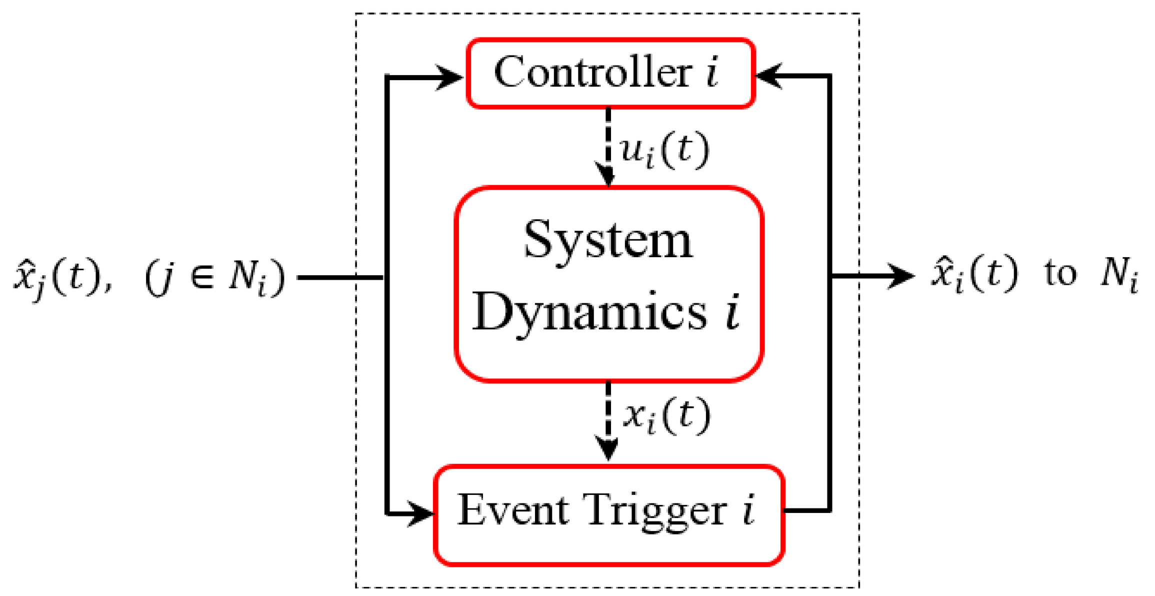
Motivation



Agent i

Motivation:

- Saving in communication for heterogeneous consensus of multi-agent networks operating in bandwidth constrained environments.
- Applying event-triggered framework to multi-agent networks.

Objectives:

- Formulate the problem as a Linear Matrix Inequality (LMI) optimization framework to compute the design parameters for the event-trigger mechanism

Problem Statement

State-space model:

$$\dot{\boldsymbol{x}}_i(t) = A\underbrace{\boldsymbol{x}_i(t)}_{\text{state}} + B_i\underbrace{\boldsymbol{u}_i(t)}_{\text{input}} + D_i\underbrace{\boldsymbol{\omega}_i(t)}_{\text{noise}}, \quad 1 \le i \le N$$

Consensus is achieved if and only if $\lim_{t\to\infty}\|\boldsymbol{x}_i(t) - \boldsymbol{x}_j(t)\| = 0, \quad (1 \le i, j \le N)$.

Event-based Control Input:

$$\boldsymbol{u}_i(t) = K_i \sum_{j \in \mathcal{N}_i} \left(\hat{\boldsymbol{x}}_i(t) - \hat{\boldsymbol{x}}_j(t)\right),$$

$\hat{\boldsymbol{x}}_i(t)$: The most recently broadcasted state of agent i;
K_i: Heterogeneous control gains to be computed.

Event-triggering function:

Transmit new state if $\boldsymbol{e}_i(t) = \hat{\boldsymbol{x}}_i(t) - \boldsymbol{x}_i(t)$ exceeds the threshold $\phi\|\hat{\mathbb{X}}_i(t)\|$ where,
$\|\hat{\mathbb{X}}_i(t)\|$: $\mathcal{N}_i\hat{\boldsymbol{x}}_i(t) - \sum_{j=1}^{\mathcal{N}_i}\hat{\boldsymbol{x}}_j(t)$,
ϕ: Transmission threshold to be computed

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Parameter Optimization

- Compute optimized unknown parameters (control gains and transmission threshold) from LMIs optimization.

$$\min_{\Theta_i,\gamma,\tau,P} \quad \gamma$$

s.t. :

$$\begin{bmatrix} \pi_{11} & \Xi\mathscr{L} & P\hat{L}_{\langle n\rangle}D & \tau M_{\langle n\rangle}^T \\ * & -\tau I & 0 & \tau M_{\langle n\rangle}^T \\ * & * & -\rho^2 I & 0 \\ * & * & * & -\gamma \end{bmatrix} < 0.$$

$$P>0, \quad \tau>0, \quad \gamma>0,$$

where

- γ, τ, P, and Θ_i $(1 \le i \le N)$ are optimization variables;
- R, ρ are given parameters for H_∞ noise reduction;
- $\hat{L}_{\langle n\rangle}$, $M_{\langle n\rangle}$, and $\mathscr{L}$ contain connectivity information;
- $\pi_{11} = A_{\langle N-1\rangle}^T P + PA_{\langle N-1\rangle} + R + \Xi\mathscr{L} + \mathscr{L}^T\Xi^T$
- $\Xi = \left(\hat{L} \otimes \mathbf{1}_n\mathbf{1}_n^T\right) \circ (\mathbf{1}_{N-1} \otimes [\Theta_1, \ldots, \Theta_N])$
- Transmission threshold and control gains are computed from:

$$\phi = \sqrt{\tau\gamma^{-1}} \qquad K_i = B_i^\dagger P^{-1}\Theta_i$$

The Consensus Algorithm

The Proposed Event-based Consensus Algorithm

Input: $\mathcal{A} = \{a_{ij}\}$, Agents' dynamics given in (1).
Output: Asymptotic Event-triggered State Consensus

Parameter Estimation: (E1 – E5)

I. *Initialization*

E1. *Transformation Matrix*: Remove N^{th} row of L in order to determine the reduced Laplacian matrix, $\hat{L}$.

E2. *System Transformation:* Determine reduced system (5).

E3. *Triggering Matrix*: Using Lemma 3, determine $M_{\langle n\rangle}$.

II. *Design*

E4. *Solving the LMI's*: Using convex optimization solvers, solve the LMIs (11) for a given $\mathcal{H}_\infty$ parameters, $\{R, \rho\}$.

E5. *Feasibility Verification:* If a solution exists for (11), obtain ϕ, and K_i's from (12). Otherwise, change parameters $\{R, \rho\}$, and repeat step E4.

Event-triggered Consensus: (C1 – C3)

C1. *Initialization*: Initialize by allowing all agents to transmit their initial states $x_i(0)$ to their neighbours.

C2. *Execution*: Using K_i's derived in Step E4, the states of agent i in (1) is excited by (2). Condition (6) is responsible to determine the next state transmission to neighbours for agent i as the states evolves to reach consensus.

C3. *Consensus Achievement*: Agent i repeats Step C2 until convergence is achieved for the disagreement state vector, i.e., $\|\hat{\mathbb{X}}_i(t)\| < \delta_i$ where δ_i is the stopping criterion.

Consensus in Heterogeneous Second-Order MAS

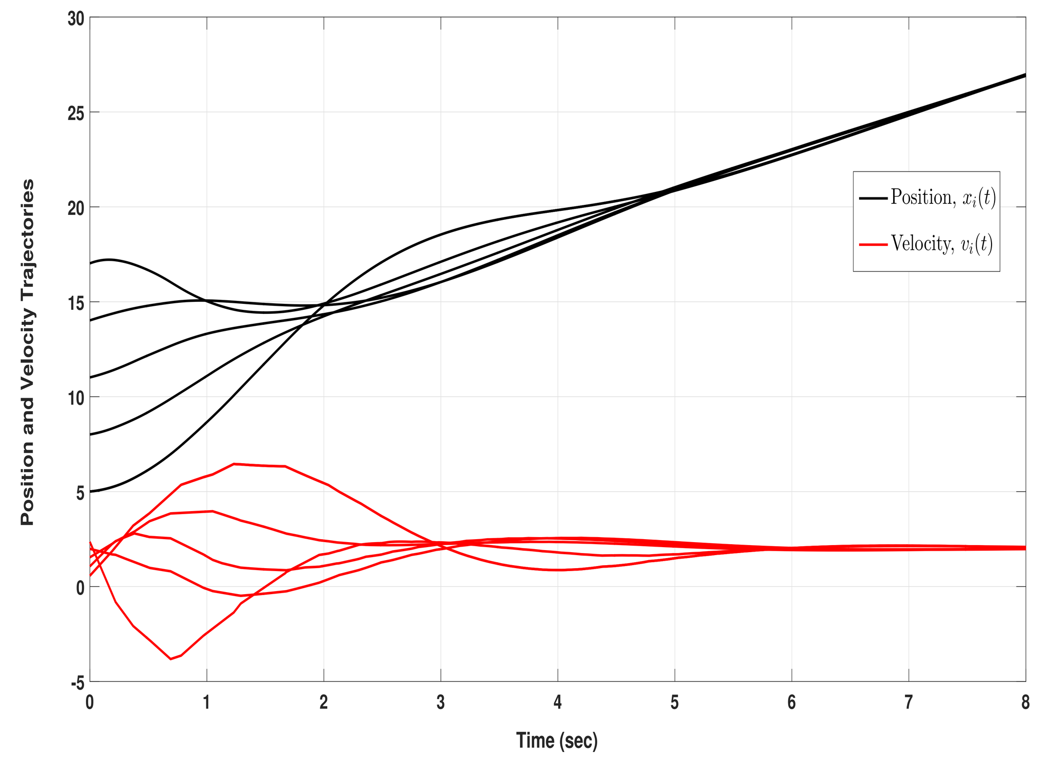

Figure 1: Evolution of state consensus

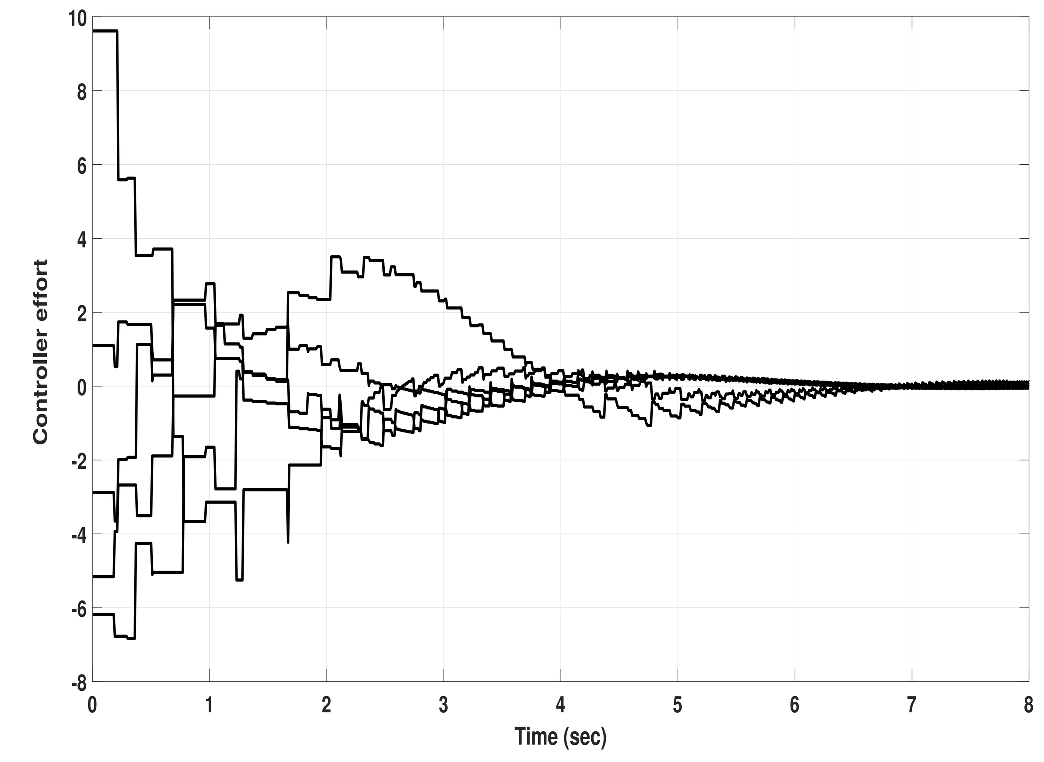

Figure 2: Controller effort $u_i(t)$.

Illustrative Example

- Five second-order mobile agents:

$$\dot{r}_i(t) = v_i(t)$$
$$m_i\dot{v}_i(t) = u_i(t) + \omega_i(t), \quad 1 \le i \le 5$$

$r_i(t)$: position, $v_i(t)$: velocity,
m_i: inertia, $\omega_i(t)$: external disturbance,

- With asymmetric connectivity Laplacian matrix:

$$L = \begin{pmatrix} 2 & -1 & 0 & -1 & 0 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & -1 & 0 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{pmatrix}.$$

Objective: All mobile agents reach the same position

- Initialization with $R=0.04I_8$, and $\rho=0.03$.
- **Optimized control gains and transmission threshold**
- $K_1=[0.50, 0.38]$, $K_2=[0.41, 0.47]$, $K_3=[0.49, 0.63]$, $K_4=[0.43, 0.46]$, $K_5=[0.74, 0.87]$, $\phi=0.212$.
- **Comparison:**

Table 1: Performance comparison for the two approaches.

Approach	# transmissions per agent					Consensus
	1	2	3	4	5	time (sec)
Proposed	39	128	119	67	123	7.32
Zhang et al.	74	106	140	88	104	8.95

- Faster consensus with a fewer number of data transmission

Conclusion

Summary:

- *Linear matrix inequality (LMI)* optimization guarantees system stability for desired *design objectives* through convex optimization
- To guarantee consensus, control gains and event-triggering condition are *coupled* to benefit from *multi-objective optimization*.
- Additional degree of freedom is provided by *designing heterogeneous control gains* for event-triggered multi-agent networks.